Indebtedness and macroeconomic imbalances in a monetary-union DSGE model

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Abstract

The euro area crisis in 2010-2012 highlighted the issue of sovereign debt in a heterogeneous monetary union. This heterogeneity concerns, in particular, inflation rates, GDP growth rates, current account balances and ratios of public deficits and debts to GDP. This paper mainly aims to investigate such macroeconomic divergences, in a DSGE model of two countries belonging to a monetary union open with the rest of the world. We first focus on divergences coming from asymmetric shocks affecting domestic production costs. Divergences are also introduced in the governments’ behavior, which may conduct pro-cyclical or counter-cyclical budgetary policy, as well as in the market perception on the default risk of the member countries. An asymmetric shock on the sovereign risk premiums is also simulated in order to discuss the transmission of financial shocks within the union. Special attention is given to two main mechanisms: the financing of government debt from banks in a risky environment and the impact of the openness of the union toward the rest of the world.

Keywords: macroeconomic divergences, euro area, DSGE, risk premium

JEL Classification: E63, F41

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1. Introduction

The euro area (EA) sovereign debt crisis in 2010-2012 highlights the issue of macroeconomic divergences across member countries. These divergences concern in particular inflation rates, GDP growth rates, ratios of public deficits and debts to GDP, and current account balances. In a monetary union with common short-term nominal interest rates but divergent national inflation rates, short-term real interest rates are different across countries. In countries where the inflation rate was above the euro area average before the financial crisis in 2008-2009, real interest rates were very low (Spain, Portugal, Italy, Greece) and even negative for short maturities (Ireland). On the contrary, in some countries with lower inflation rates, such as Germany, real interest rates were higher. As a consequence, indebtedness of the private sector rose and boosted aggregate demand in the former group of countries while domestic consumption and investment rose more slowly in the latter group. Over time, the former group was bound to lose price competitiveness while the other group was likely to gain price competitiveness. One could expect that net exports of goods and services would decrease and, as a result, GDP growth would slow down in the first group of countries while net exports would increase and GDP growth would be higher in the second group of countries.

Thus, a competitiveness channel (divergent real exchange rates) would correct imbalances due to an indebtedness channel (divergent real interest rates). However, the indebtedness channel has been predominant for too much a long time, not only because it was aggravated by public deficits, but also because the competitiveness channel takes time to work given that the response of trade volumes to relative prices is weak or with a lag. Moreover, due to growing current account imbalances, the required adjustments in real exchange rates would be large: a real depreciation was needed in countries with high current account deficits and a real appreciation was needed in countries with current account surpluses. As regards intra-zone real exchange rates, prices and wages should grow slower in high-deficit countries and higher in countries with external surpluses. However, since 2008, the extent of adjustment has mostly relied on the high-deficit countries. As a consequence, the deflationary policies have been harmful for Southern populations (in Greece, Spain or Portugal). As regards extra-zone real exchange rates, high-deficit countries cannot rely on the effective nominal depreciation of the euro. Indeed, the exchange rate of the euro depends mainly on financial flows with the rest of the world (admittedly there have been net outflows since 2011 and an effective nominal and real depreciation of the euro). Anyway, an effective real depreciation of the euro would not help reducing euro area macroeconomic imbalances because it would increase current account surpluses in surplus countries in the same time as it would decrease current account deficits in deficit countries (Guillemette and Turner, 2013).

A credit channel may also be a cause of macroeconomic divergences among EA countries. At the beginning of the financial crisis, when the European Central Bank (ECB) lowered its key interest rates, the transmission to interest rates on loans could be incomplete in countries where banks had weak financial positions. The supply of loans could even decrease, which would be the opposite effects of what was expected by the central bank. In such a case, aggregate spending would grow more slowly in these countries.
Macroeconomic divergences across EA countries might also stem from fiscal imbalances. Before the crisis, the public sector could borrow heavily as did the private sector in countries where real interest rates have been lower since the entry into the euro area. As a consequence, public saving decreased (in Greece and Portugal) and public deficits and debts have not been lowered as much as what was set in the fiscal rules of the Stability and Growth Pact (SGP). In some countries (Italy, France), the ratio of public debt to GDP has not risen much despite some persistent public deficits because they benefited from a context of low interest rates. In most other EA countries, the ratio of public debt to GDP was decreasing over the 1999-2007 period. Hence, the current issue of public indebtedness has been caused by the financial crisis. The bail-out of the banking sector led to a sharp increase in the GDP-public debt ratio (Ireland) and the recession led to an increase in public expenditure and a decrease in tax revenues more or less in EA countries depending on the size of automatic stabilizers and the extent of discretionary measures. Interest expenditure grew faster in countries where the risk premium had increased much. A higher default risk premium could be explained by a high level of public debt, fast rising public indebtedness or bad future prospects of economic growth. Contagion effects affected the long-term interest rates in other EA countries whose banks were highly exposed to the public bonds of high-debt countries or had lent much to the banking sector of the latter countries.4

Macroeconomic performance of individual countries in the euro area may diverge because of national differences in economic structures or in economic policies. Various economic and financial interdependencies among member countries – what we call transmission channels – are also a source of divergence. This paper aims to investigate trade, credit and fiscal channels of macroeconomic divergences across EA countries (the last two channels being financial channels). To do so, we build a dynamic stochastic general equilibrium (DSGE) model of two countries belonging to an economic and monetary union (EMU). The main original features of our model are the following: i) EMU is open to the rest of the world (ROW); ii) there is public indebtedness (with a constraint on debt accumulation) and external debt (the net foreign asset position of the nation); iii) there are deviations from the law of one price within EMU and between EMU and the ROW; iv) there is a financial accelerator mechanism; v) banks lend to firms and to each government of the two EMU countries; and vi) there are three endogenous risk premia in the model, namely a firm risk premium depending on the net worth-capital value ratio, a government risk premium depending on the public debt/GDP ratio, and a nation-wide risk premium depending on the net foreign asset (NFA) position.

Our model also shares some common features with those of other DSGE models of the euro area. There are both real rigidities (habit formation in consumption and adjustment costs in investment) and nominal rigidities (Calvo probability of not being able to reset prices) as in Smets and Wouters (2002), Coenen, McAdam and Straub (2008), Moyen and Sahuc (2008), Erceg and Lindé (2012). Each government in EMU countries finances public purchases (consumption and transfers) by levying taxes (on consumption and capital income) and issuing debt. We ignore seigniorage and taxes on labour services and wages unlike Coenen, McAdam and Straub (2008). Following Christoffel, Jaccard and Kilponen (2011), we assume that each government adjusts the fiscal instrument (public

4 For instance, Cyprus asked for financial support because domestic banks were suffering from losses in bad loans to Greek banks.
consumption in our model) with reaction to output growth and the level of debt (in deviation with its steady-state level). We add some degree of inertia in the adjustment of fiscal instrument.

In our model, the trade channel works through both intra-EA and extra-EA change in net exports and bilateral real exchange rates. The credit channel concerns the transmission of the common monetary policy to domestic production and inflation in both EA countries via the banking sector’s balance sheet. The share of consumers’ deposits that banks lend to borrowers depends on economic activity. As for the fiscal channel, it denotes the fiscal causes of macroeconomic divergences across EA countries. Focusing on the role of indebtedness of both public and private sectors and on the role of risk premia, we show how the fiscal channel can be closely related to the credit channel. In particular, the interaction relies on the exposure of banks to public debt via their holding of securities issued by general government of EA countries and on the consequences of higher public indebtedness on the bank lending to the economy (firms). We also study the fiscal channel by introducing some asymmetries between countries. Besides the effects of asymmetric shocks, we look at the impact of national disparities with regard to the value of some parameters, such as the elasticity of government risk premium with regard to the level of public debt. We add an asymmetry regarding the stance of public spending over the business cycle: the latter is procyclical in one country and countercyclical in the other.

In what follows, we make a review of the literature on macroeconomic divergences in the euro area and the literature on DSGE models of EA countries with either a focus on fiscal policy or a focus on risk premia (section 2). We then set the DSGE model of a two-country EMU (section 3), and describe the calibration and simulations (section 4). We finally explain our results (section 5) and conclude (section 6).

## 2. Literature review

We propose to make a short review of two strands of the literature on the euro area. The first strand deals with the sources of macroeconomic divergences across EA countries. The second strand is about the features of a micro-founded open-economy DSGE model applied to the euro area and used to analyze various policy experiments or the effect of shocks.

In the empirical model built by Angeloni and Ehrmann (2004), inflation differentials are mainly caused by asymmetries in inflation persistence across EA countries. In Deroose et al (2004), cyclical divergence is analyzed by simulating the effects of asymmetric shocks. Van den Noord (2004) builds a small simple model in order to illustrate cyclical divergence between big and small EA countries in the early years of EMU. In his model, divergence may come from three different channels: a real interest rate channel, a real exchange rate channel and a real house price channel. Cyclical divergence is due to a greater exposure of small countries to shocks, given a stronger housing channel in these countries. Westaway (2003) develops a three-country NK model (the UK, the EA and the rest of the world) and shows that inflation differentials are in fact an adjustment mechanism to asymmetric shocks inside EMU. In particular, if prices are more flexible in the UK than in the EA, then the UK would suffer from higher inflation volatility (but lower output volatility) if the country were inside the monetary union. Indeed, the real exchange rate would have to adjust through relative price
changes and the variation of the common nominal interest rate induced by asymmetric shocks would not necessarily be suitable to the British economy.

Coenen, McAdam and Straub (2008) explain the features of the new area wide model (NAWM) of the euro area which is used for macroeconomic projections by the ECB. The home country is the EA and the foreign country is the US. In this micro-founded open-economy DSGE model of the euro area, there are habits in consumption and adjustment costs in investment and imports among others. There are heterogeneous households: the unconstrained household holds bonds and accumulates physical capital while the constrained household has no access to financial markets and holds money. There is local-currency pricing and imperfect exchange rate pass-through. And there is a risk premium on internationally traded bonds which depends on the NFA position of the domestic country relative to domestic output. They apply the NAWM to fiscal issues. In their model, the fiscal authority levies various tax rates on consumption, wage income and capital income. They calibrate and simulate the model in order to investigate the effects of reducing tax rates on hours worked and output in the EA. They also show the spillover effects of such tax reductions on the main trade partner. However, the NAWM cannot be used for studying asymmetries across EA member countries, because the euro area is modeled as a global area or as a single big country. Gomes, Jacquinot and Pisani (2010) develop and calibrate a four-country version of the NAWM of the EA: the EAGLE (Euro Area and Global Economy) model, with two (identical) euro area countries (Germany and the rest of union) and two countries outside the euro area (the U.S. and the rest of the world). With such a model, cross-country spillovers originating from domestic or foreign shocks can be studied. In their model, public debt is issued only on domestic financial markets and the financial sector is not explicitly modeled.

Vogel, Roeger and Herz (2012) investigate various fiscal policy rules in a DSGE model of a small country in a closed monetary union (where the law of one price holds). Fiscal instruments react with a one-period lag to the terms of trade (given that price competitiveness influences output). The welfare effects of fiscal policy depend not only on the choice of the fiscal instrument (purchases, transfers or taxes) and the budgetary closure rule (lump-sum taxes, transfers or distortionary taxes are alternatively used to stabilize debt), but also on the type of shocks (productivity or risk premium shock) and the type of households (liquidity constrained households or unconstrained households).

Coenen, Mohr and Straub (2008) use the NAWM of the euro area in order to study the effects of fiscal consolidation aiming at lowering the level of public debt. They show that the long-term effects are positive because the decline in interest payments makes it possible to decrease distortionary taxes. However, the short-term effects are negative (lower consumption of the constrained household) and differ depending on the composition of fiscal consolidation. Erceg and Lindé (2012) compare in a two-country closed monetary union DSGE model the effects of fiscal consolidation based on spending cuts or tax hikes in a context where monetary policy is constrained by the zero lower bound (ZLB) on nominal interest rates. In their model, there are rule-of-thumb consumers that consume all of their after-tax income. They also introduce a financial sector. In order to minimize output losses over time, they propose a mixed strategy of front-loaded tax hikes and deferred expenditure cuts.
Christoffel, Jaccard and Kilponen (2011) estimate a closed-economy DSGE model (using U.S. data) in order to find the main determinants of government bond risk premia. Since it is assumed that the government issued long-term default-free bonds, the risk premium is interpreted as compensation either for the risk of a capital loss in case of selling the bond before maturity or for the risk of erosion of the bond’s value due to inflation. They show that procyclical public expenditure leads to a higher bond premium. In a closed economy DSGE model, Corsetti et al (2012) focus on a “sovereign risk channel” through which a higher government bond risk premium spills over to the borrowing cost of the private sector. Under the ZLB constraint, the central bank cannot cut interest rates further. As a result, the sovereign risk premium amplifies macroeconomic fluctuations. In the closed economy DSGE model of Bi (2012), the sovereign risk premium is endogenously determined: it rises if the government approaches the “fiscal limit”, that is the maximum level of debt that the government is able or willing to service. The latter depends on either economic fundamentals (the economy is on the slippery side of the Laffer curve) or the government willingness to raise taxes. Roeger and in’t Veld (2013) extends the analysis of the sovereign risk channel in the euro area within a two-country closed monetary union DSGE model. The sovereign risk channel relies on the vulnerability of banks to a decline in bond prices. They compare the contractionary effects of fiscal consolidation with the effects of no-consolidation in a context of rising public debt.

In our model, we put the focus on the stance of public primary spending over the business cycle by comparing the implications of a countercyclical vs. procyclical reaction of public primary expenditure to deviations of output from its steady-state level. Procyclical spending makes it harder to stabilize debt to the level that prevailed before a given shock. We also study how a higher risk premium on public bonds issued by one EMU country may be beneficial to the other EMU country as long as the latter may enjoy lower risk premium just because risks are perceived higher elsewhere.

3. Model Overview

We build a three countries DSGE model: two symmetric countries (home \((h)\) and foreign\((f)\)) of equal size are members of a monetary union and are open to the rest of the world \((w)\) which is fully exogenous (in the small open economy perspective).

The model contains price stickiness, monopolistic competition in final goods market, capital adjustment costs, incomplete pass-through of exchange rate via law of one price deviation, financial market frictions and fiscal policy instruments.

Each economy is populated by households, banks, government and three types of producers: entrepreneurs, capital producers, and retailers (domestic and imported goods retailers). There is a common monetary authority that sets the unique nominal risk-free interest rate for both countries.

Capital producers build new capital and sell it to the entrepreneurs. Entrepreneurs produce wholesale goods and sell them to domestic goods retailers. Domestic and imported goods retailers set nominal prices of final goods à la Calvo (1983). Banks convert households’ deposits in loans to finance the government deficit and the entrepreneurial purchase of capital. Each government decides upon fiscal policy.

This model has specific features that distinguish it from standard models of monetary union. Indeed, the existence of banks which can lend to entrepreneurs, to domestic and foreign governments, allows
us to describe the fiscal mechanism of macroeconomic divergences within monetary union (or the sovereign risk channel in the literature). The fiscal channel relies on the vulnerability of domestic banks to public indebtedness in each country.

In the financial channel, the model captures the fact that the financial vulnerability of one country in the monetary union can lead to changes in the financing cost for the other country. The exposure of domestic banks to foreign government debt allows us to study the potential contagion effects working through sovereign debts within the monetary union. Finally, the consideration of intra and extra-zone net exports for each member state makes the model useful to explain the importance of trade openness in times of regional economic turmoil. The law of one price gap and incomplete pass-through allow us to draw inflation and real interest rate divergences within the monetary union.

3.1. Households

Each country $i \in \{h, f\}$ is populated by a continuum of unit mass households with infinite life. The representative household of country $i$ maximizes the following expected discounted sum of utilities:

$$E_t \sum_{t=0}^{\infty} \beta^t \left( \frac{(C^i_t - hC^i_{t-1})^{1-\sigma}}{1-\sigma} - \frac{(N^i_t)^{1+\eta}}{1+\eta} \right)$$

(1)

where $C^i_t$ is the aggregate consumption and $N^i_t$ denotes the number of hours worked. $E_t$ is the conditional expectation operator. The parameters $0 < \beta < 1$, $\sigma > 0$, $\eta > 0$ and $0 < h < 1$ are, respectively, the subjective discount factor, the inverse intertemporal elasticity of substitution, the inverse of the Frisch elasticity of labor supply and the parameter that controls the habit persistence.

In each country $i \in \{h, f\}$ of the union, the household’s period-by-period budget constraint is defined by:

$$(1 + \tau^i_{t,t})C^i_t + \frac{D^i_t}{p^i_t} + \frac{S^i_tB^i_{W,t}}{p^i_t} = \frac{W^i_t}{p^i_t}N^i_t + R^i_{t-1} \frac{D^i_{t-1}}{p^i_{t-1}} + R^i_{W,t-1} \Psi^i_{b,t-1}(b^i_{t-1}, Z^i_{t-1}) \frac{S^i_tB^i_{W,t-1}}{p^i_t} + \frac{TR^i_t}{p^i_t} + \Lambda^i_t$$

(2)

where $p^i_t$ is the consumer price index (CPI), $W^i_t$ is the nominal wage, $D^i_t$ is nominal deposits that pay gross nominal interest rate $R^i_t$ and $B^i_{W,t}$ is nominal internationally traded bonds, denominated in rest of the world currency, that pay a gross nominal interest rate $R^i_{W,t-1}\Psi^i_{b,t-1}; S^i_t$ is the nominal exchange rate (expressed in terms of units of home currency per unit of foreign currency). $\tau^i_{t,t}$, $TR^i_t$ and $\Lambda^i_t$ are, respectively, distortionary tax on consumption, government transfers and real profits from the monopolistic sector. Finally, in the budget constraint, $\Psi^i_{b,t}$ represents a risk premium that is a function of the economy’s real aggregate level of net-foreign asset position in percentage of steady-state output, as follows:

$$\Psi^i_{b,t}(b^i_t, Z^i_t) = \exp \left( - \psi^i_b \left( \frac{S^i_tB^i_{W,t}}{Y^i_p t} \right) + Z^i_t \right)$$

(3)
where \( b_t \equiv \frac{s_t B_{t-1}}{y P_t} \) is real aggregate net-foreign asset position in percentage of steady-state GDP; \( \psi_b > 0 \) is a measure of the elasticity of the risk premium with respect to net-foreign asset position, and \( Z_{t-1} \) is an exogenous shock on risk premium defined by \( \log(Z_{t-1}) = \rho_z \log(Z_{t-1}) + \varepsilon_{z,t} \) with \( \varepsilon_{z,t} \sim i.i.d. (0, \sigma_z^2) \).

The term \( \Psi_{b,t}(b_t, Z_t) \) is assumed to be strictly decreasing in \( b_t \) and to satisfy \( \Psi_b(0,0) = 1 \). It captures imperfect integration in the international financial markets and ensures a well-defined steady-state in the model (Schmitt-Grohé and Uribe (2003)).

Households choose the paths for \( C_t^i, N_t^i, D_t^i, B_{w,t}^i \) in order to maximize (1) subject to the budget constraint in (2). The following optimality conditions hold:

\[
(C_t^i - hC_{t-1})^{1-\sigma} - \beta h E_t(C_{t+1}^i - hC_t)^{1-\sigma} = \lambda_t^i (1 + \tau _{t,t})
\]

\[
\frac{\lambda_t^i W_t}{P_t} = (N_t^i) \eta
\]

\[-\frac{\lambda_t^i S_t}{P_t} + E_t \beta \frac{\lambda_{t+1}^i S_{t+1}}{P_{t+1}} R_{w,t} \Psi_{b,t}(b_t, Z_t) = 0
\]

\[-\frac{\lambda_t^i I_t}{P_t} + E_t \beta \frac{\lambda_{t+1}^i I_{t+1}}{P_{t+1}} R_t = 0
\]

\( \lambda_t^i \) is the Lagrangian multiplier in (4), (5), (6) and (7).

The final good, \( X_t^i \), in each country \( i \in \{h, f\} \) of monetary union, which is allocated to consumption, \( C_t^i \), investment, \( I_t^i \), and public spending, \( G_t^i \), is an aggregate function of goods produced in the home country, \( X_{h,t}^i \), goods produced in foreign country (rest of monetary union), \( X_{f,t}^i \), and goods produced in the rest of the world, \( X_{w,t}^i \):

\[
X_t^i = \left[ (1 - a_1^i - a_2^i) \frac{1}{\bar{\theta}} \left( X_{h,t}^i \right)^{\frac{\theta - 1}{\bar{\theta}}} + (a_1^i) \frac{1}{\bar{\theta}} \left( X_{f,t}^i \right)^{\frac{\theta - 1}{\bar{\theta}}} + (a_2^i) \frac{1}{\bar{\theta}} \left( X_{w,t}^i \right)^{\frac{\theta - 1}{\bar{\theta}}} \right]^{\frac{\bar{\theta}}{\theta - 1}}
\]

for \( X \{C, I, G\}; i, k \in \{h, f\} \) and \( i \neq k \).

The parameters \( \theta > 1, a_1^i, \) and \( a_2^i \) are, respectively, the elasticity of substitution between the three types of goods, the share of imported goods from the rest of union and the share of imported goods from the rest of world. We suppose that these shares are identical reciprocally between each country \( i \in \{h, f\} \) of the union and the rest of world. The fraction \( 1 - a_1^i - a_2^i \) therefore is the degree of home bias in consumption, investment and public goods.

The price index (CPI) associated to (8) is given by:

\[
P_t^i = \left[ (1 - a_1^i - a_2^i) \left( P_{i,t}^i \right)^{1-\theta} + a_1^i \left( P_{h,t}^i \right)^{1-\theta} + a_2^i \left( P_{w,t}^i \right)^{1-\theta} \right]^{1/(1-\theta)}
\]

We define \( X_{h,t}^i \equiv \left( \int_0^1 X_{h,t}^i(j) \frac{X}{X} dj \right)^{\frac{X}{X-1}} \), \( X_{f,t}^i \equiv \left( \int_0^1 X_{f,t}^i(j) \frac{X}{X} dj \right)^{\frac{X}{X-1}} \) and
\[ X_{w,t}^i = \left( \int_0^1 X_{w,t}^i(j) \frac{x d_j}{X_{w,t}^i(j)} \right)^{\frac{1}{\gamma-1}} \] as the composite aggregates of differentiated varieties produced domestically, inside and outside of the monetary union, respectively, with \( \gamma \) being the elasticity of substitution between varieties originating in the same country; \( X_{w,t}^i(j) \), \( X_{k,t}^i(j) \) and \( X_{w,t}^i(j) \) being a typical variety \( j \) of domestic goods, imported goods from foreign country and imported goods from the rest of the world, respectively. The corresponding prices are deduced easily and are given by, respectively:

\[
P_{i,t}^i = \left( \int_0^1 p_{i,t}^i(j) \frac{1-x d_j}{p_{i,t}^i(j)} \right)^{\frac{1}{\gamma-1}}, \quad P_{k,t}^i = \left( \int_0^1 p_{k,t}^i(j) \frac{1-x d_j}{p_{k,t}^i(j)} \right)^{\frac{1}{\gamma-1}}, \quad P_{w,t}^i = \left( \int_0^1 p_{w,t}^i(j) \frac{1-x d_j}{p_{w,t}^i(j)} \right)^{\frac{1}{\gamma-1}},
\]

where \( P_{i,t}^i(j) \) (respectively \( P_{k,t}^i(j) \) and \( P_{w,t}^i(j) \) ) is the price of a typical variety \( j \) produced in the home country (respectively imported prices from the rest of the union and the rest of the world).

The optimal demands for domestic, foreign and rest of the world goods, issued from expenditure minimization\(^5\), are:

\[
x_{i,t}^i = \left( 1 - a_1^i - a_2^i \right) \left( \frac{P_{i,t}^i}{P_{i,t}^i(j)} \right)^{\theta} X_{i,t}^i
\]

(10)

\[
x_{k,t}^i = a_1^i \left( \frac{P_{k,t}^i}{P_{k,t}^i(j)} \right)^{\theta} X_{k,t}^i
\]

(11)

\[
x_{w,t}^i = a_2^i \left( \frac{P_{w,t}^i}{P_{w,t}^i(j)} \right)^{\theta} X_{w,t}^i
\]

(12)

All these relations hold symmetrically for \( i, k \in \{h, f\} \) and \( i \neq k \).

### 3.2. Open Economy Relations

This section outlines the key relations that describe the terms of trade, the real exchange rates and the law of one price deviations.

For each country \( i, k \in \{h, f\} \) and \( i \neq k \), we define the bilateral terms of trade as:

\[
TOT_{i,t}^k = \left( \frac{P_{i,t}^i}{P_{i,t}^k(j)} \right)^{\theta} \quad \text{and} \quad TOT_{i,t}^w = \left( \frac{P_{i,t}^i}{P_{w,t}^i(j)} \right)^{\theta}
\]

(13)

where \( P_{i,t}^i \) and \( P_{w,t}^i \) are, respectively, the domestic price of imported goods from the rest of the union (foreign country) and from the rest of the world (ROW). \( P_{i,t}^i \) is the domestic price of home goods.

From (9), the terms of trade can be related to the CPI-PPI ratio as follows:

\[
\frac{P_{i,t}^i}{P_{i,t}^i(j)} = \left[ \left( 1 - a_1^i - a_2^i \right) + a_1^i \left( TOT_{i,t}^k \right)^{1-\theta} + a_2^i \left( TOT_{i,t}^w \right)^{1-\theta} \right]^{\frac{1}{1-\theta}}
\]

(14)

\(^5\)The optimization program is: \( \min_c \theta \theta\epsilon_{c,t} \epsilon_{w,t} c_t \epsilon_{c,t} p_{i,t}^i C_{i,t}^i + P_{k,t}^i C_{k,t}^i + P_{w,t}^i C_{w,t}^i = P_{i,t}^i C_{i,t}^i \) subject to the following constraint:

\[
C_{i,t}^i = \left[ \left( 1 - a_1 - a_2 \right) \hat{\theta}(C_{i,t}^i)^{\theta-1} + \left( a_1 \right) \hat{\theta}(C_{k,t}^i)^{\theta-1} + \left( a_2 \right) \hat{\theta}(C_{w,t}^i)^{\theta-1} \right]^{\frac{1}{\theta}}.
\]
In this paper, we assume that the law of one price (LOP) holds for the export sector, but there is incomplete pass-through in the import sector. This assumption is motivated by the existence of monopolistic domestic importers in the union that practice local currency pricing (Devereux and Engel (2001)). This behaviour can make the price of the foreign goods in the domestic market temporarily deviate from the producer price level in the country of origin. The wedge between these two prices is called the law of one price gap (LOPG) and is given by, bilaterally:

$$LOPG_{k,t}^i = \frac{p_{k,t}^i}{p_{k,t}^i} \text{ and } LOPG_{w,t}^i = \frac{s_t p_{w,t}^i}{p_{w,t}^i}$$

(15)

where $p_{k,t}^i$ and $p_{w,t}^i$ are domestic prices in country $k$ of the region and the rest of the world.

Similarly, we define the bilateral real exchange rates as follows:

$$RER_{k,t}^i = \frac{p_{k,t}^i}{p_t^i} \text{ and } RER_{w,t}^i = \frac{s_t p_{w,t}^i}{p_t^i}$$

(16)

Finally, we can express the effective terms of trade, the effective law of one price gap and the effective real exchange, for each country $i \in \{h, f\}$ as:

$$TOT_i^i = (TOT_{k,t}^i)^{a_1}(TOT_{w,t}^i)^{a_2}$$

(17)

$$LOPG_i^i = (LOPG_{k,t}^i)^{a_1}(LOPG_{w,t}^i)^{a_2}$$

(18)

$$RER_i^i = (RER_{k,t}^i)^{a_1}(RER_{w,t}^i)^{a_2}$$

(19)

Assuming that the two countries $\{h, f\}$ are of the same size in the monetary union, the effective real exchange for the union is therefore:

$$RER_u^u = (RER_h^h)^{\frac{1}{2}}(RER_f^f)^{\frac{1}{2}} = (RER_h^h)^{\frac{a_1}{2}}(RER_f^f)^{\frac{a_2}{2}}$$

(20)

which can be written also in terms of euro nominal exchange rate:

$$RER_t^u = \frac{s_t p_{t}^w}{p_{t}^u}$$

(21)

where $P_{t}^u$ and $P_{t}^w$ are CPI of the monetary union and the rest of the world.

### 3.3. Production sector

#### 3.3.1. Entrepreneurs

The entrepreneurs play an important role here because their presence allows us to introduce the financial accelerator mechanism.

In each country $i \in \{h, f\}$, as in Bernanke et al. (1999), entrepreneurs manage a continuum of firms $j \in [0,1]$ that produces, by using $K_t^i$ units of capital and $N_t^i$ units of labor, wholesale (intermediate) goods in a perfect competitive market according to the following technology:

$$Y_t^i(j) = A_t^i K_t^i(j)^{\alpha} N_t^i(j)^{1-\alpha}$$

(22)
where $A_t^i$ is a technological shock that is common to all firms and follows a stationary first-order autoregressive process: $\log(A_t^i) = \rho A_t^i \log(A_{t-1}^i) + e_{A,t}^i$, with $e_{A,t}^i \sim i.i.d.(0, \sigma^2_A)$; $\alpha \in [0,1]$ is the share of capital in the production technology.

The representative firm maximizes its profit by choosing $K_t^i$ and $N_t^i$ subject to the production function (22). The first-order conditions for this optimization problem are:

$$w_t^i = (1 - \alpha)mc_t^i \frac{y_t^i}{t_{Lt}^i}$$

$$mpc_t^i = \alpha mc_t^i \frac{y_t^i}{K_t^i}$$

(23)

(24)

where $mc_t^i$ is the Lagrangian multiplier associated with the production function (22) and denotes the real marginal cost; $w_t^i$ is the real wage; and $mpc_t^i$ is the real marginal productivity of capital.

Entrepreneurs are risk neutral and borrow to finance a share of capital used in the production process. As in Bernanke et al. (1999), to ensure that they never accumulate enough funds to fully self-finance their own activities entirely, we assume they have a finite expected horizon. In each period $t$, entrepreneurs face a constant probability $(1 - \nu)$ of leaving the economy. We follow Christensen and Dib (2008) in allowing newly entering entrepreneurs to inherit a fraction of the net worth of those firms who exit from the business. This assumption is made in order to ensure that new entrepreneurs start out with a positive net worth. In contrast, Bernanke et al. (1999) ensure this by assuming that entrepreneurs also work. This difference does not affect the results.

At the end of each period, entrepreneurs purchase capital, $K_{t+1}^i$, that will be used in the next period at the real price $q_t^i$. Thus, the total funding needed by an entrepreneur to purchase capital is $q_t^i K_{t+1}^i$. The capital acquisition is financed partly by their net worth, $NW_{t+1}^i$, and by borrowing, $q_t^i K_{t+1}^i - NW_{t+1}^i$, from a financial intermediary. Financial intermediaries (banks) obtain their funds from household deposits. Their activities are described below (see section 2.4.).

In optimum, the entrepreneur’s aggregate demand for capital in the economy depends on the expected marginal return and the expected marginal financing cost at $t + 1$. Thus, the capital demand must satisfy the following differentiation between the ex post marginal return on capital, $E_t(R_{K,t+1}^i)$, and the marginal productivity of capital at $t + 1$, $mpc_t^i$, which is the rental rate of capital:

$$E_t(R_{K,t+1}^i) = E_t \left[ (1 - \tau_t^i) \frac{mpc_t^i}{q_t^i} + (1 - \delta)q_{t+1}^i \right]$$

(25)

where $\delta$ is the capital depreciation rate, $\tau_t^i$ is the tax rate on capital-income (whose introduction here is a specific feature of our model) and $(1 - \delta)q_{t+1}^i$ is the value of one unit of capital used in $t + 1$.

Following Bernanke et al. (1999), we assume the existence of an agency problem that makes external finance more expensive than internal finance, because financial intermediaries are facing costs for auditing the performance of entrepreneurs. However, entrepreneurs observe the random outcome of their investments costlessly and decide whether to repay their debt or to default. If they default, the lenders audit the project and seize whatever they find. As demonstrated in Bernanke et al. (1999), the
optimal financial contract between borrower and lender implies an external finance premium (the difference between the cost of external and internal finance), \( \Psi_{t+1}^i \), that reflects the existence of auditing costs and depends on the entrepreneur’s leverage ratio (capital to net worth ratio).

Accordingly, the entrepreneur’s demand for capital satisfies, optimally, the equality between expected return on capital and gross premium for external finance plus the gross real opportunity costs equivalent to the gross real interest rate on loans:

\[
E_t \left( R^i_{K,t+1} \right) = E_t \left[ \Psi_{E,t+1}^i \left( \frac{R^i_{K,t}}{\pi^i_{t+1}} \right) \right]
\]

(26)

where \( R^i_{K,t} \) is the gross nominal interest rate on banks loans; \( \Psi_{E,t+1}^i \) is the function that describes how the external finance premium depends on the financial position of the firm and is given by:

\[
\Psi_{E,t+1}^i (\cdot) = \left( \frac{NW^i_{t+1}}{q^i_{t+1}} \right)^{-\gamma} \psi_{E,i}^i(1) = 1 \text{ and } \gamma \text{ is the elasticity of the external finance premium with respect to firm’s leverage ratio. Thus, the external finance premium is an equilibrium inverse function of the aggregate financial position in the economy, expressed by the leverage ratio. Equation (26) provides the basis for the financial accelerator. If entrepreneur’s net worth goes up, the external finance premium falls, the cost of borrowing falls and firms get cheaper access to credit.}

Aggregate entrepreneurial net worth accumulation of the economy depends on profits earned in previous periods plus the bequest \( \Omega^i_t \), that newly entering entrepreneurs receive from entrepreneurs who leave the economy, and evolves according to:

\[
NW^i_{t+1} = \nu \left[ R^i_{K,t} q^i_{t-1} K^i_t - R^i_{L,t-1} \left( \frac{NW^i_t}{q^i_{t-1}} \right)^{-\gamma} \left( q^i_{t-1} K^i_t - NW^i_t \right) \right] + (1 - \nu) \Omega^i_t
\]

(27)

### 3.3.2. Capital producers

Competitive capital producers use a linear technology to produce new capital \( K^i_{t+1} \) from final investment goods \( I^i_t \) and existing capital stock leasing from entrepreneurs without costs. When producing capital, they are subject to quadratic capital adjustment costs specified as

\[
\frac{\psi_{I}}{2} \left( \frac{I^i_t}{K^i_t} - \delta \right)^2 K^i_t.
\]

The aggregate capital stock used by producers in each economy \( i \) evolves as follow:

\[
K^i_{t+1} = \left[ \frac{I^i_t}{K^i_t} - \psi_{I} \left( \frac{I^i_t}{K^i_t} - \delta \right)^2 \right] K^i_t + (1 - \delta) K^i_t
\]

(28)

where \( \psi_{I} > 0 \) is the parameter that measures the adjustment costs elasticity.

Capital producers face an optimization problem which consists, in real terms, in choosing the level of investment that maximizes their profits:

---

\( ^6 \) For details, see Bernanke et al. (1999).
The following equilibrium condition holds:

\[
q_t - \psi_t \left( \frac{I_t^i}{K_t^i} - \delta \right) = 1
\]

which is the standard Tobin’s Q equation that links the price of capital to the marginal adjustment costs.

When \( \psi_t = 0 \) (in absence of adjustment costs), the capital price, \( q_t^i \), is constant and equal to 1. This shows that capital adjustment costs imply necessarily the capital price (\( q_t^i \)) variation and therefore contribute to the volatility of entrepreneurial net worth.

### 3.3.3. Retailers: price and inflation dynamics

The presence of retailers provides the source of nominal stickiness in the economy. In each country \( i \in \{ h, f \} \), all retailers take wholesale goods as inputs, repackage these costlessly, and sell them in a monopolistically competitive market. There are domestic goods retailers and imported goods retailers. Following Calvo (1983), we assume that retailers set nominal prices on a staggered basis: each period, a fraction \( 1 - \phi^i_t \) of retailers are randomly selected to set new prices while the remaining fraction \( \phi^i_t \) of retailers keep their prices unchanged. For simplicity, these fractions are assumed to be equal within the two groups of retailers.

All home goods retailers purchase the wholesale goods from entrepreneurs at a price equal to the entrepreneurs’ nominal marginal cost. Each retailer \( j \) of them setting price at \( t \) will choose the optimal price, \( \bar{P}^i_{t,t} \), that maximizes the expected profits for \( s \) periods, so that:

\[
\max_{\bar{P}^i_{t,t}(j)} \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} \left( \beta \phi^i_t \right)^s \frac{\lambda^i_{t+s}}{\lambda^i_t} [Y^i_{t,t+s}(j)(\bar{P}^i_{t,t}(j) - P^i_{t,t+s}mc^i_{t,s})] \right\}
\]

subject to the demand function,

\[
Y^i_{t,t+s}(j) = \left( \frac{P^i_{t,t+s}(j)}{\phi^i_{t,t+s}} \right)^{1-\chi} Y^i_{t,t+s}, \quad \text{where} \quad \frac{\lambda^i_{t+s}}{\lambda^i_t} \text{ is the households’ marginal utilities ratio between } t + s \text{ and } t.
\]

The first-order condition for this problem yields,

\[
\bar{P}^i_{t,t}(j) = \frac{X \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} (\beta \phi^i_t)^s \frac{\lambda^i_{t+s}}{\lambda^i_t} Y^i_{t,t+s}(j) P^i_{t,t+s}mc^i_{t,s} \right\}}{X \mathbb{E}_t \left\{ \sum_{s=0}^{\infty} (\beta \phi^i_t)^s \frac{\lambda^i_{t+s}}{\lambda^i_t} Y^i_{t,t+s}(j) \right\}}
\]

Aggregating across all retailers, the price index for domestically produced goods is given by,

\[
P^i_{t,t} = \left[ (1 - \phi^i_t)(\bar{P}^i_{t,t})^{1-\chi} + \phi^i_t(P^i_{t,t-1})^{1-\chi} \right]^{1/(1-\chi)}
\]
Combining log-linearized versions of equations (32) and (33) yields an expression for the gross inflation rate for domestically produced goods, defined by the following New Keynesian Phillips curve:

$$\hat{\pi}_{t+1}^{\nu} = \beta E_t \hat{\pi}_{t+1}^{\nu} + \frac{(1 - \phi^i)(1 - \beta \phi^i)}{\phi^i} \hat{m}c_t^i$$

where $mc_t^i$ is the real marginal cost, $\pi_{t,t}^i = \frac{p_{t,t}^i}{p_{t,t-1}^i}$ is gross domestic inflation and variables with hats are log deviations from the steady-state values.

Similarly, for each economy $i, k \in \{h, f\}$ with $i \neq k$, imported goods retailers purchase the products from foreign producers at the wholesale price, $P_{t,t}^i$. At the wholesale level, the law of one price holds. Thus, $P_{t,t}^i = P_{k,t}^i$ and $P_{t,t}^i = S_tP_{w,t}^w$ are the wholesale prices (nominal marginal costs) for goods coming from foreign country and the rest of the world, respectively. But at the retail level, we assume that the law of one price does not hold (such as $P_{k,t}^i \neq P_{k,t}^k$ and $P_{t,t}^i \neq S_tP_{w,t}^w$), which introduces the effect of the incomplete exchange rate pass-through into the model. Similar to the home good retailers, imported goods retailers set prices according to a Calvo-style price setting equation. Their optimization problems are identical except the real marginal costs that differentiate them. The real marginal costs are, respectively, $(p_{t,t}^k) \equiv LOCPG_{k,t}^i$ and $(S_tP_{w,t}^w) \equiv LOCPG_{w,t}^i$ for imported goods from the rest of the monetary union and the rest of the world. The inflation rates for imported goods then satisfy these following New Keynesian Phillips curves:

$$\hat{\pi}_{k,t} = \beta E_t \hat{\pi}_{k,t+1} + \frac{(1 - \phi^i)(1 - \beta \phi^i)}{\phi^i} \hat{lop}g_{k,t}^i$$

$$\hat{\pi}_{w,t} = \beta E_t \hat{\pi}_{w,t+1} + \frac{(1 - \phi^i)(1 - \beta \phi^i)}{\phi^i} \hat{lop}g_{w,t}^i$$

where $\pi_{k,t}^i$ and $\pi_{w,t}^i$ are imported inflation prices from foreign country and the rest of the world.

Finally, from equation (9), CPI inflation, $\hat{\pi}_t^i$, is a composite of domestic, foreign and world goods prices inflation, such that:

$$\hat{\pi}_t^i = \hat{\pi}_t^{\nu} + \hat{\pi}_t^{k} + \hat{\pi}_t^{w}$$

$$\forall i, k \in \{h, f\} \text{ and } i \neq k.$$
interest. Following Atta-Mensah and Dib (2008), we assume that the intermediation process $\varepsilon^i_t$ is partly endogenous and depends on the state of the economy in this way:

$$
\varepsilon^i_t = \left( \frac{Y^i_t}{Y^i} \right)^\epsilon Z^i_{e,t}
$$

(39)

The parameter $\epsilon$ is the elasticity of the willingness to lend with respect to changes in economic activity (deviations of output from its steady-state value) and $Z^i_{e,t}$ represents shocks to the intermediation process. Assume that the willingness to lend is procyclical, then $\epsilon > 0$. This can be justified by the fact that, in good times, net worth of entrepreneurs and governments’ fiscal receipts are relatively high. This improves the credit risks of borrowers and increases the willingness of financial intermediaries to lend. The process for $Z^i_{e,t}$ is given by:

$$
\log(Z^i_{e,t}) = \rho_\epsilon \log(Z^i_{e,t-1}) + \epsilon^i_{t,t}
$$

(40)

The shock $Z^i_{e,t}$ could represent for example an exogenous change in the confidence level of bank with respect to the credit risks of their borrowers and the health of the economy. Next, we assume that banks provide the fixed shares of their loans to entrepreneurs, foreign government and domestic government. For each economy $i,k \in \{h,f\}$ with $i \neq k$, these shares are, respectively, $\zeta^i_h, \zeta^i_f$ and $(1 - \zeta^i_h - \zeta^i_f)$.

At the end of period $t$, the representative bank receives, in principal and interest, $R^i_{L,t} \Psi^i_{E,t} (\cdot) (\zeta^i_h R^i_{L,t})$ from entrepreneur, $R^i_{L,t} \Psi^i_{E,t} (i^k_t, i^k_t, Z^i_{e,t}) (\zeta^i_f R^i_{L,t})$ from foreign government and $R^i_{L,t} (1 - \zeta^i_h - \zeta^i_f) R^i_{L,t}$ from domestic government. We assumed that domestic government loan cost is the benchmark for bank to decide which costs must be applied on the others agents’ loans. Bank makes therefore loans consistent with a portfolio management goal and $R^i_{L,t}$ is the gross nominal interest rate on domestic government loans; Hence, the relative risk premia are defined as follow: $\Psi^i_{E,t} (\cdot)$ is entrepreneur’s external finance premium as previously defined and $\Psi^i_{E,t} (i^k_t, i^k_t, Z^i_{e,t})$, defined in section 2.5., is the relative risk premium paid by government $k$. The bank owes $R^i_{L,t}$ to its depositors and earns a zero net return on its reserves. In this case, the bank profit function is given by:

$$
\Pi^i_{L,t} = R^i_{L,t} (1 - \zeta^i_h - \zeta^i_f) L^i_t + R^i_{L,t} \Psi^i_{E,t} (\cdot) (\zeta^i_h L^i_t) + R^i_{L,t} \Psi^i_{E,t} (\cdot) (\zeta^i_f L^i_t) + D^i_t - R^i_{L,t} (1 - \zeta^i_h - \zeta^i_f) L^i_t - \zeta^i_h L^i_t - \zeta^i_f L^i_t
$$

(41)

Given the competition among banks for loans and deposits, the zero profit condition guarantees that:

$$
R^i_{L,t} = \frac{R^i_t - 1 + \varepsilon^i_t}{\left[ 1 + \zeta^i_h (\Psi^i_{E,t} - 1) + \zeta^i_f (\Psi^i_{E,t} - 1) \right] \varepsilon^i_t}
$$

(42)

and the fluctuations in the reserve levels of banks would be reflected in the gap between loan and deposit interest rates. Equation (42) shows that the loans interest rate applied to domestic government decreases when risk premia of relative risky agents increases, and thus when their debts increase.

---

7 The others possible sources of variation in $Z^i_{e,t}$ are: perceived changes in entrepreneur’s cash flow or net worth, government regulation of banks, technological advances in the intermediation process (Berger (2003)).
3.5. Government Budget Constraint and Fiscal policy

In each economy \(i, k \in \{h, f\} \) with \(i \neq k\), government spends in purchases of aggregate goods \(G_t^i\) and transfers to households \(TR_t^i\). To do that, the government collects tax revenues on consumption and capital, receives the loans from domestic and foreign banks \((1 - \zeta_k^i - \zeta_k^k) L_t^i \) and \(\zeta_k^k L_t^k\) respectively.

The government \((i, k \in \{h, f\} \) with \(i \neq k\) ) budget constraint is given by:

\[
(1 - \zeta_k^i - \zeta_k^k) L_t^i + \zeta_k^k L_t^k = R_{L,t-1}^i(1 - \zeta_k^i - \zeta_k^k) L_t^i + R_{L,t-1}^i \Psi_{L,t}^i(l_t^i, l_t^k, Z_{l,t}) \zeta_k^k L_t^k + PD_t^i \tag{43}
\]

In equation (43), \(PD_t^i\) is the nominal primary deficit and expressed by:

\[
PD_t^i = P_t^i G_t^i + TR_t^i - \tau_{C,t} P_t^i C_t^i - \tau_{K,t} mp c_t K_t^i P_t^i q_t^i \tag{44}
\]

and \(\Psi_{L,t}(l_t^i, l_t^k, Z_{l,t})\) is the government’s relative risk premium such that,

\[
\Psi_{L,t}(l_t^i, l_t^k, Z_{l,t}) \equiv \exp \left( \psi_i \left( \frac{(1 - \zeta_k^i - \zeta_k^k) L_t^i + \zeta_k^k L_t^k}{Y_t^i P_t^i} \right) + Z_{l,t} \right) \tag{45}
\]

where \(\psi_i^l\) is the elasticity of the risk premium with respect to government debt; \(l_t^i \equiv \frac{l_t^i}{Y_t^i P_t^i}\) and \(l_t^k \equiv \frac{l_t^k}{Y_t^i P_t^i}\) are respectively the total of loans made by banks in economy \(i\) and \(k\); \(Z_{l,t}\) is exogenous shock on government’s premium and evolves according to following autoregressive process:

\[
\log(Z_{l,t}) = \rho_l \log(Z_{l,t-1}) + \epsilon_{l,t} \tag{46}
\]

Fiscal policy instruments:

The government needs to adjust tax revenues or expenditure to stabilize its deficit and debt. Government spending adjustments in response to cyclical fluctuations are endogenously made according to this fiscal rule:

\[
\log \left( \frac{G_t^i}{G_t^i} \right) = \rho_g \log \left( \frac{G_{t-1}^i}{G_t^i} \right) - (cg) (1 - \rho_g) \rho_{gy} \log \left( \frac{Y_t^i}{Y_t^i} \right) - (1 - \rho_g) \rho_{gl} \log \left( \frac{DY_t^i}{DY_t^i} \right) + \epsilon_{g,t} \tag{47}
\]

where \(\rho_g, \rho_{gy}, \rho_{gl} \in [0,1]\) capture, respectively, the degree of public spending smoothing, fiscal reaction to output deviation and fiscal reaction to debt/GDP ratio \((DY_t^i \equiv \frac{(1 - \zeta_k^i - \zeta_k^k) L_t^i + \zeta_k^k L_t^k}{Y_t^i P_t^i})\); \(\epsilon_{g,t}\) is exogenous shock to government spending \((\epsilon_{g,t} ~ i. i. d. (0, \sigma_{e_g}^2))\).

The parameter \((cg)\) captures the degree of fiscal policy cyclical. If \((cg) = 1\) (resp. \((cg) = -1\) ), public spending is counter-cyclical (resp. procyclical).

As the other fiscal instruments, government transfers, taxes on consumption and capital follow the autoregressive process, such as:

\[
\log \left( \frac{TR_t^i}{TR_t^i} \right) = \rho_{tr} \log \left( \frac{TR_{t-1}^i}{TR_t^i} \right) + \epsilon_{tr,t} \tag{48}
\]

\[
\log \left( \frac{\tau_{c,t}^i}{\tau_c} \right) = \rho_{tc} \log \left( \frac{\tau_{c,t}^i}{\tau_c} \right) + \epsilon_{tc,t} \tag{49}
\]

\[
\text{where } \rho_g, \rho_{gy}, \rho_{gl} \in [0,1]; \epsilon_{g,t} \sim \text{i.i.d.} (0, \sigma_{e_g}^2).
\]
\[ \log\left(\frac{\tau_{k,t}}{\tau_{k}}\right) = \rho_{xk} \log\left(\frac{\tau_{k,t}}{\tau_{k}}\right) + e_{\tau k,t} \] (50)

where, \( \rho_x \in [0,1] \), with \( x = tr, tc \) and \( \tau K \), are the coefficients of autoregressive process and \( e_{x,t} \sim i.i.d.\left(0,\sigma_{e_x}^2\right) \) are the associated exogenous shocks.

### 3.6. Monetary authority

In the monetary union, the common central bank sets the nominal interest rate according to the following Taylor-type interest rate rule:

\[ \log\left(\frac{R_t}{R}\right) = \beta_0 \log\left(\frac{R_{t-1}}{R}\right) + (1 - \beta_0) \left[ \beta_1 \log\left(\frac{\pi_t^{um}}{\pi_t^{um}}\right) + \beta_2 \log\left(\frac{Y_t^{um}}{Y_t^{um}}\right) \right] + e_{r,t} \] (51)

with \( e_{r,t} \sim i.i.d.\left(0,\sigma_{e_r}^2\right) \).

\( R, \pi^{um} \) and \( Y^{um} \) are the steady-state values of \( R_t, \pi_t^{um} \) and of \( Y_t^{um} \), that are, respectively, the nominal interest rate, the inflation and the output of the union. \( \pi_t^{um} \) and \( Y_t^{um} \) are the average values of inflation and output of the two equal size countries:

\[ \pi_t^{um} = \frac{1}{2}(\pi_t^h + \pi_t^f) \quad \text{and} \quad Y_t^{um} = \frac{1}{2}(Y_t^h + Y_t^f) \] (52)

\( \beta_1 > 1 \) and \( \beta_2 < 1 \) are coefficients that measure central bank responses to expected inflation and output deviations. \( 0 < \beta_0 < 1 \) captures the degree of interest rate smoothing by monetary authority.

### 3.7. General Equilibrium conditions

In equilibrium, the factor markets, the final goods market, the loan market and the international traded bonds market must clear in each country \( i \in \{h,f\} \).

Equilibrium in factor markets requires:

\[ N_i^t = \int_0^1 N_i(j)dj \quad \text{and} \quad K_i^t = \int_0^1 K_i(j)dj \] (53)

The loan market clears when the unused fraction of household deposits in reserves by financial intermediaries equalizes the total funds lent to entrepreneurs, domestic government and foreign government:

\[ L_i^t = E_i^t D_i^t \] (54)

Let \( Y_t^i \equiv \left(\int_0^1 Y_t^i(j)\frac{x-1}{x}d_j\right)^{\frac{1}{x-1}} \) denote aggregate output. Thus, the goods market clearing condition satisfies:

\[ Y_t^i = C_t^i + I_t^i + G_t^i + EX_t^i \] (55)

where \( EX_t^i = a_i^1 \left(\frac{p_i^t}{P_k^t}\right)^{-\theta} A_k^t + a_i^2 \left(\frac{p_i^t}{S_t P_k^t}\right)^{-\theta} A_w^t \) is total exports.

Then the domestic economy’s aggregate resource constraint can be rewritten as:

\[ \forall i,k \in \{h,f\} \text{ and } i \neq k, \]
\[ Y_t^i = \left( \frac{p_{i,t}}{P_t} \right)^{-\theta} \left[ (1 - a_1^i - a_2^i)AB_t^i + a_1^i \left( \frac{1}{RER_{k,t}} \right)^{-\theta} AB_t^k + a_2^i \left( \frac{1}{RER_{w,t}} \right)^{-\theta} AB_{w,t} \right] \] (56)

where \( AB_t^i, AB_t^k \) and \( AB_{w,t}^i \) are, respectively, domestic, rest of the union and the rest of the world absorptions such that:

\[ AB_t^i = C_t^i + I_t^i + G_t^i \] (57)

\[ AB_t^k = C_t^k + I_t^k + G_t^k \] (58)

\( AB_{w,t} \) is an exogenous process.

The internationally traded bonds market is in equilibrium when the positions of the export and importing firms vis-à-vis the rest of the world equals the households’ choice of internationally traded bonds holdings.

For \( i, k \in \{ h, f \} \) and \( i \neq k \), the evolution of net foreign assets (government assets holdings from the rest of the union plus households internationally traded bonds holdings) at the aggregate level can be expressed as:

\[ S_t B_{w,t} + \zeta_t^k L_t^k = S_t R_{w,t-1} W_{b,t-1} (b_{t-1}^i, Z_{t-1}^i) B_{w,t-1} + R_{t-1}^i W_{l,t-1} (t_{t-1}, t_{t-1}, Z_{t-1}^i) \zeta_t^k L_t^k + EX_t^i \]

\[ - (I_{M,t}^i + I_{M,w,t}) \] (59)

where \( I_{M,t}^i \) and \( I_{M,w,t} \) are imports country \( i \) originating from country \( k \) and from the rest of the world, respectively.

Noting that the definitions of \( b_t^i, l_t^i \) and \( L_t^k \) are: \( b_t^i \equiv \frac{S_t B_{w,t}}{Y_{t}} \), \( l_t^i \equiv \frac{L_t^i}{Y_{t}} \) and \( L_t^k \equiv \frac{L_t^k}{Y_{t}} \), we can write the evolution of total real net foreign assets position in percentage of GDP as:

\[ b_t^i + \zeta_t^k L_t^k \frac{p_t^k}{p_t^i} = \frac{R_{w,t-1} W_{b,t-1}}{\pi_t} b_{t-1}^i + \frac{R_{t-1} W_{l,t-1}}{\pi_t} \zeta_{t-1}^k L_{t-1}^k \frac{p_{t-1}^k}{p_{t-1}^i} + \frac{1}{Y_t} (\frac{p_t^i}{p_t^i} Y_t - C_t - I_t - G_t) \] (60)

\( \forall i, k \in \{ h, f \} \) and \( i \neq k \).

3.8. Rest of the world

We assume that the rest of the world is fully exogenous and its variables follow the autoregressive process such that:

\[ \log(AB_{w}^w) = \rho_{Aw} \log(AB_{w-1}^w) + e_{Aw,t} \] (61)

\[ \log(R_{w,t}) = \rho_{rw} \log(R_{w,t-1}) + e_{rw,t} \] (62)

\[ \log(\pi_{w,t}) = \rho_{\pi w} \log(\pi_{w,t-1}) + e_{\pi w,t} \] (63)

where, \( \rho_x \in [0, 1] \) with \( x = ABw, Rw \) and \( \pi w \) are the coefficients of autoregressive process and \( e_{x,t} \sim i.i.d(0, \sigma_x^2) \) are the associated exogenous shocks.

4. Calibration and simulations

The calibration of our model and the main macroeconomic ratios at their steady-state are summarized in Table 1, as well as the references used for the parameterization.
<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inverse intertemporal elasticity of substitution</td>
<td>$\sigma$</td>
<td>2</td>
<td>AL</td>
</tr>
<tr>
<td>Inverse of the Frisch elasticity of labor supply</td>
<td>$\eta$</td>
<td>1</td>
<td>AL</td>
</tr>
<tr>
<td>Subjective discount factor</td>
<td>$\beta$</td>
<td>0.99</td>
<td>AL</td>
</tr>
<tr>
<td>Habit persistence coefficient</td>
<td>$h$</td>
<td>0.85</td>
<td>AL</td>
</tr>
<tr>
<td>Share of imported goods from the rest of the union</td>
<td>$a_1^i$</td>
<td>0.21</td>
<td>AC</td>
</tr>
<tr>
<td>Share of imported goods from the rest of the world</td>
<td>$a_2^i$</td>
<td>0.11</td>
<td>AC</td>
</tr>
<tr>
<td>Elasticity of substitution between domestic and imported goods</td>
<td>$\theta$</td>
<td>1.5</td>
<td>Coenen &amp; al. (2008)</td>
</tr>
<tr>
<td>Elasticity of the risk premium respect to net-foreign asset position</td>
<td>$\psi_i^j$</td>
<td>0.001</td>
<td>Schmitt-Grohé &amp; Uribe (2003)</td>
</tr>
<tr>
<td>Capital contribution to production</td>
<td>$\alpha$</td>
<td>0.36</td>
<td>AC</td>
</tr>
<tr>
<td>Capital depreciation rate</td>
<td>$\delta$</td>
<td>0.025</td>
<td>AL</td>
</tr>
<tr>
<td>Internal capital adjustment costs parameter</td>
<td>$\psi_i^o$</td>
<td>0.25</td>
<td>AL</td>
</tr>
<tr>
<td>Fraction $\phi_i$ of retailers keeping their prices unchanged</td>
<td>$\phi_i$</td>
<td>0.8</td>
<td>AL</td>
</tr>
<tr>
<td>Elasticity of the external finance premium with respect to firm’s leverage ratio</td>
<td>$\gamma$</td>
<td>1</td>
<td>AL</td>
</tr>
<tr>
<td>Entrepreneurs probability of leaving the economy $i$</td>
<td>$1 - \nu$</td>
<td>0.272</td>
<td>Bernanke &amp; al. (1999)</td>
</tr>
<tr>
<td>Share of banking loans to entrepreneurs</td>
<td>$\zeta_i^r$</td>
<td>0.65</td>
<td>AC</td>
</tr>
<tr>
<td>Share of banking loans to foreign government</td>
<td>$\zeta_i^k$</td>
<td>0.195</td>
<td>AC</td>
</tr>
<tr>
<td>Elasticity of risk premium with respect to government debt</td>
<td>$\psi_i^l$</td>
<td>0.001</td>
<td>Coenen &amp; al. (2008)</td>
</tr>
<tr>
<td>Elasticity of the willingness to lend with respect to changes in economic activity</td>
<td>$\epsilon$</td>
<td>1.82</td>
<td>AC</td>
</tr>
</tbody>
</table>

**Steady State Macroeconomic Ratios**

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital / GDP ratio</td>
<td>$K / Y$</td>
<td>8</td>
</tr>
<tr>
<td>Consumption/GDP ratio</td>
<td>$C / Y$</td>
<td>0.6</td>
</tr>
<tr>
<td>Investment/GDP ratio</td>
<td>$I / Y$</td>
<td>0.2</td>
</tr>
<tr>
<td>Public expenditures/GDP ratio</td>
<td>$G / Y$</td>
<td>0.2</td>
</tr>
<tr>
<td>Transfers/GDP ratio</td>
<td>$Tr / Y$</td>
<td>0.13</td>
</tr>
</tbody>
</table>

**Monetary and fiscal policy**

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smoothing coefficient in the monetary rule</td>
<td>$\beta_0$</td>
<td>0.8</td>
<td>AL</td>
</tr>
<tr>
<td>Inflation stabilizing coefficient in the monetary rule</td>
<td>$\beta_1$</td>
<td>2</td>
<td>AL</td>
</tr>
<tr>
<td>Output stabilizing coefficient in the monetary rule</td>
<td>$\beta_2$</td>
<td>0.1</td>
<td>AL</td>
</tr>
<tr>
<td>Smoothing coefficient in the public expenditure rule</td>
<td>$\rho_g^0$</td>
<td>0.9</td>
<td>Coenen &amp; al. (2008)</td>
</tr>
<tr>
<td>Output stabilizing coefficient in the public expenditure rule</td>
<td>$\rho_{gy}$</td>
<td>0.3</td>
<td>Christoffel &amp; al. (2011)</td>
</tr>
<tr>
<td>Debt stabilizing coefficient in the public expenditure rule</td>
<td>$\rho_{gl}$</td>
<td>0.01</td>
<td>Christoffel &amp; al. (2011)</td>
</tr>
</tbody>
</table>

**Shocks**

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficients of autoregressive process for technology</td>
<td>$\rho_a$</td>
<td>0.6</td>
</tr>
<tr>
<td>Coefficients of autoregressive process for shock on government’s premium</td>
<td>$\rho_l$</td>
<td>0.5</td>
</tr>
</tbody>
</table>

*Note: AC- Authors’ calculations, AL- average values in the literature.*
It is made according to the references found in the literature for the euro area and/or to the authors’ computation based on the European Central Bank and European Commission databases. First of all, we assume that the two member-countries in the monetary union are symmetric. Only the Governments’ default risk premiums may differ in some simulations, in order to depict potential pessimistic expectations of the financial markets about the sovereign debt sustainability in one country of the monetary union. Asymmetries are also assumed in the Governments’ behavior with regard to the conduct of fiscal policy. Thus, the coefficient of reaction of public expenditures to the output-gap is either negative for a countercyclical fiscal policy, or positive in the case of a procyclical fiscal policy. Since one Government always adopts a counter-cyclical behavior (being disciplined), the second one may be less disciplined and decide to use a pro-cyclical fiscal rule, instead of a counter-cyclical one, in periods of economic growth.

In what follows, the model is subject to two types of shocks: 1) an asymmetric technological shock that affects one country of the union, implying a reduction in the production costs, and 2) an asymmetric default risk premium shock, in line with the idea of pessimistic expectations of the financial markets about the sovereign debt sustainability in one country of the monetary union. In the rest of the paper, we present in more depth the reaction of the domestic economy affected by shocks as well as the transmission of shocks to the other member country of the union. Special attention will be given to two mechanisms: the financing of government debt from banks in a risky environment and the impact of the openness of the union toward the rest of the world.

5. Results

We start the presentation of our results by the simulation of the asymmetric technological shock, before analyzing the shock on the sovereign risk premium. We assume a counter-cyclical fiscal policy as the baseline situation for all shocks. To address the question of the fiscal discipline in a monetary union, we compare this baseline situation to a pro-cyclical action of one Government in the case of a technological shock that generates transitory? economic growth.

5.1 Technological shock analysis

The impulse response functions depicted in this paragraph correspond to a technological positive shock affecting only one country of the union, called hereafter domestic country. The shock causes a temporary decrease in the production costs that stimulates output and creates the premises for economic growth in this country. Figure 1 gives a comparative description of the economic dynamics for domestic country and for the other member of the union, respectively. The black continuous line makes reference to the domestic economy hit by the shock, while the red dashed line depicts the reaction of the foreign country to the shock.

For the domestic economy, we recognize the classical impact of a supply shock: higher economic growth, higher investment, consumption and lower inflation compared to the steady state level. As a consequence, the domestic country becomes more competitive within the union and with regards to the RoW simultaneously. Indeed, the effective real exchange rate depreciates, on the one side because of the lower domestic inflation and on the other side because of the nominal depreciation of
the common currency. Since the rental rate of capital decreases considerably in the economy, the Government revenues reduce (because of fewer tax receipts on capital) and the ratio of public debt on GDP is increasing despite lower public expenditures (which follow a counter-cyclical rule). Debt rises due to increased interest payments? Or because the decline in tax receipts is larger than the decline in public consumption?

We can also notice on Figure 1 that the domestic technological shock has little impact on the other members to the union (see the foreign country). Almost all variables are close to their steady-state level. The positive effect on the output is explained by the gain of external competitiveness with the rest of the world, due to the nominal depreciation of the common currency, partly compensated by the loss of competitiveness within the union.

**Figure 1. Comparative impact of a technological shock on domestic vs. foreign economy**

Since the technological shock generates economic growth in the domestic economy, we use it hereafter to analyze the implications of fiscal discipline or indiscipline of some members within the union. We thus associate a disciplined behavior to a counter-cyclical public expenditure rule and an undisciplined behavior to a pro-cyclical one. Indeed, when the domestic country is disciplined, it will

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8 We note that a more persistent shock may lead to an initial appreciation of the effective real exchange rate and a loss of external competitiveness for domestic country which could explain a lower output growth and investment just after the shock.
reduce public expenditure in period of economic growth to create budget surpluses necessary in periods of downturn. Under the undisciplined behavior, domestic government decides to stimulate even more the economic growth by increasing the public expenditures after the shock. Figure 2 and 3 present the comparative simulations for the domestic and foreign country respectively.

When the public authority has a pro-cyclical behavior in the country hit by the shock, the nominal depreciation of the common currency is amplified. That’s why the depreciation of the effective real exchange rate is stronger simultaneously in the domestic and foreign country (figures 2 and 3). The gains from the trade openness to the rest of the world explain the higher level of output under this scenario, for all the members of the monetary union. Cumulated to the increase of public expenditure, the additional external demand for domestic goods limits the fall of inflation and of the rental rate of capital after the technological shock. This explains why the pro-cyclical budgetary policy is unexpectedly not followed by a decline of the Debt to GDP ratio. Public expenditure goes up, but budgetary receipts will increase even more. In turn, the lower primary deficit reduces the public debt/GDP expansion in the short-run. In the long-run, the accumulation of debt may become dangerous. In the last graph of figure 2, the public debt/GDP ratio does not return easily to the steady state under this scenario.

![Figure 2. Fiscal policy comparison under technological shock (domestic economy)](image-url)
For the foreign country (figure 3), the fiscal indiscipline of the other member of the union (see the previous domestic country) lowers national demand. The falls in consumption and investment are amplified by fiscal indiscipline, because of an increase in the real interest rate. Under a single monetary policy, the low inflation (mainly explained by the imported inflation from the other country of the union) conducts to a high real interest rate that discourages national demand.

The additional external demand coming from the rest of the world suppresses the negative impact of the shock on the national demand and explains the amelioration of the rental rate of capital in this country. The increase in the fiscal revenues coming from taxes and the fall of counter-cyclical public expenditure conduct to a reduction of the public debt/GDP ratio, as depicted in the last graph of figure 3.

*Note: the counter or pro-cyclical public expenditures make reference to the other country of the union (directly affected by the technological shock). The foreign economy simply receives indirectly (by contagion) the effect of the shock and/or fiscal indiscipline of the other member.

**Figure 3. Fiscal policy comparison under technological shock (foreign economy)**

The lesson to be formulated for the euro area starting from these simulations is the following. Before implementing restrictive budgetary policy, in periods of quite low economic growth (in order to improve the health of public finances) governments may attentively consider the effects on the real economy. In an open economy, the public spending reduction may generate appreciation of the effective real exchange rate, loss of competitiveness and consequently fall in the rental rate of capital and in the fiscal revenues. The loss of fiscal revenues may be higher than the reduction in public
expenditure. The health of public finances becomes worse in that case, contrary to the original objective of the public authority, at least in the short-run.

5.2 Sovereign risk premium analysis

We assume in this paragraph that one country of the union is hit by a shock that adjusts its sovereign risk premium upward. This situation may be assimilated to an unexpected overestimation of the sovereign risk of some European countries. Indeed, after a long period where all European countries benefited from the same financial conditions despite their economic and structural asymmetries, with the recent financial crisis, the financial markets overreacted to the public debt accumulation, applying excessive risk premiums to Greece, Portugal, Spain or Italy. Because the market perception about the European countries sovereign risk changed after the crisis, we cannot consider it only temporary. That is why we simultaneously introduce a permanent source of heterogeneity, namely in the sensitivity coefficient of the sovereign risk premium to the public debt/GDP ratio. This coefficient is supposed to be very close to zero (0.001) for the low-risk country and higher (0.1) for the high-risk country hit by the shock. The impulse response functions of the main macroeconomic variables are depicted in Figure 4, comparatively for the two member countries of the union.

![Figure 4. Comparative impact of a risk premium shock on domestic vs. foreign economy](image-url)
An increase in the risk premium of the domestic country (hit by the shock) makes it relatively riskier compared to the other country of the union. As a consequence, the optimal behavior of national banks implies higher interest rate for domestic government bonds, and implicitly for the financing of the real economy. Even if the specific risk associated to entrepreneurs decreases relative to the national government bonds (see the lower external finance premium for firms compared to the steady state level), the cost of credits goes up. Investment decreases and inflation goes down. The real interest rates goes up, explaining the downward adjustment of consumption. Beyond the low level of domestic demand, the output-gap is positive. Since the national government is supposed to have a counter-cyclical behavior, the economic growth may only come from the openness of the economy, namely from the rest of the world. These adjustments are related to the depreciation of the effective real exchange rate in reaction to the risk premium shock, implying higher foreign demand for domestic goods. This explains in the same time the investment dynamics, see the relative small decrease despite the higher real interest rate in the economy. All in all, it appears that the shock on the risk premium has negative impact on the domestic country, but this effect is lower when the degree of openness toward the rest of the world is high. The model could thus explain some of the recent dynamics of the most risky European countries. Interest expenditure grew faster in countries where the risk premium had increased much. These phenomena affected not only the public sector but also the private sector, via a fiscal channel, which relies on the vulnerability of banks to public indebtedness. The costs of external finance went up in the economy, limiting investments and prospects of economic growth, particularly in countries with low export capacity. Unfortunately, since the world growth was slowing down, European countries could not export more to the rest of the world.

Looking on the reaction of the foreign country to the increase of the risk premium for the other member of the union (see the previous domestic economy), we observe a beneficial impact on it. We can easily distinguish two sources of economic growth: an internal one and another one related to the openness of the economy to international trade. To better explain them, let’s recall the behavior of national banks facing the higher risk associated to the foreign government bonds. For them, the national sovereign bonds become safer investment when compared to the foreign public bonds. The national Government thus benefits from lower costs to finance its debt, having also positive effect of the financial conditions for all domestic agents. Real interest rates go down, stimulating consumption and investment in this country, which are sources of internal growth. The depreciation of the common currency explains in the same time the depreciation of the effective real exchange rate and the gain of external competitiveness for this country. For high export capacity, this external source of growth may be very important. We could recall this kind of mechanisms to explain, for example, the strong economic growth in Germany in the recent post-crisis period.

6. Conclusion

This paper aims to investigate macroeconomic divergences, in a DSGE model of two countries belonging to a monetary union open with the rest of the world. It first focuses on divergences coming from asymmetric shocks affecting domestic production costs. Divergences are also introduced in the governments’ behavior, which may conduct pro-cyclical or counter-cyclical budgetary policy. We
find that a counter-cyclical budgetary policy in order to improve the health of public finance is not necessarily suitable in an open monetary union. It may generate appreciation of the effective real exchange rate, loss of competitiveness and consequently fall in the rental rate of capital and in the fiscal revenues. The loss of fiscal revenues may be higher than the reduction in public expenditure. The health of public finances becomes worse in that case, contrary to the original objective of the public authority, at least in the short-run.

An asymmetric shock on the sovereign risk premiums is simulated in order to discuss the transmission of financial shocks within the union. Special attention is given to two main mechanisms in that case: the financing of government debt from banks in a risky environment and the impact of the openness of the union toward the rest of the world. A shock on the risk premium has negative impact on the domestic economy, but this effect is lower when the degree of openness toward the rest of the world is high. For the rest of the monetary union, the increase in the default risk perception for one country is beneficial. Benefits are growing with the degree of openness of each economy to the rest of the world. The model is thus able to explain, for example, the economic downturn during the sovereign debt crisis in countries with low export capacity, or the strong economic growth in Germany in the recent post-crisis period.

Future researches based on this model will be oriented toward the study of the public debt management in a heterogeneous monetary union, with either sovereign debt entirely managed at a national level, or at the union-wide level (by the use of euro-bonds instead of the national governmental bonds).

References


