Correlated Bank Runs*

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Abstract

This article extends the application of global games of Goldstein and Pauzner (2005) in the banking model of Diamond and Dybvig (1983) to account for correlation in the quality of banks' long term investment, when banks are linked through cross deposits and there is a central bank. The goal is to study how these elements affect the deposit contract that banks offer to depositors and the ex ante probability of a bank run. We show that the coexistence of a central bank, which determines banks' reserve requirements, and an interbank market, which redistributes reserves, leads to a smaller probability of a bank run and to less inefficient bank runs, relative to the case with no central bank and no interbank market. By adequately choosing the level of reserves to store, the central bank can improve the equilibrium outcome and allow banks to offer a higher interim payment to depositors, relative to the situation with no cross deposits.

Keywords: Bank Runs; Correlated Investment; Interbank Market

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1 Introduction

The risk of bank runs is an inevitable feature of any banking system. Banks take short term deposits from lenders and make longer term investments. Since the pioneering work of Diamond and Dybvig in 1983, it is well understood that this « maturity transformation » is arguably the key function of a bank. On the downside, it brings with it the risk that depositors may ask for their money in large numbers at a time when the bank does not have the liquid resources to meet these demands.

This paper focuses on three elements of bank runs in reality, which combined are less well-understood. The first element is the fact that information about the quality of banks' long-run investments is not perfect. The second element is the fact that the quality of banks' investments may be correlated. The third one is the fact that banks exchange cross deposits through the interbank market.

The collapse of large financial institutions observed during the early stages of the global financial crisis of 2007-2012 illustrates the importance of imperfect information and correlation of banks' investment strategies. In this regard, in a June 2008 speech, U.S. Treasury Secretary Timothy Geithner, then President and CEO of the New York Federal Reserve Bank, referred to the freezing of credit markets observed some months before and placed significant blame on the « run » of entities in the « parallel » banking system, being engaged in the same type of investment strategies, for which financial innovation had made it difficult to evaluate the quality of their investments.[1]

This paper extends the application of global games of Goldstein and Pauzner (2005) in the banking model of Diamond and Dybvig (1983) to examine how, in the presence of an interbank market and a central bank, imperfect information about the quality of banks' long-run investments, which can be correlated, affects the deposit contract that banks offer to depositors and the *ex-ante* probability of a bank run.

As in Goldstein and Pauzner (2005), we show that there is a unique Bayesian equilibrium, in which a bank run occurs if the quality of banks' long-run investments is below some threshold. What is nonetheless specific to our paper is that we demonstrate that the adequate interaction between the central bank and the interbank market can lead to a smaller probability of a bank run and to less inefficient bank runs, relative to the case with no central bank and no interbank market.[2]

In our model, there are three periods, three regions and three banks. Each bank has a disjoint set of depositors. In the initial period, the central bank, common to all regions, determines the

[1] The importance of correlation has also been stressed by Acharya and Yorulmazer (2007) and Farhi and Tirole (2011), who theoretically show that banks have incentives to correlate their risk in the expectation of systemic bailouts.

[2] Wang (2012) has already added the government to the Goldstein and Pauzner (2005) model, to study how the announcement of bailouts affects the probability of a bank run.

fraction of deposits that banks must store as reserves, if they choose to participate in the interbank market. After observing this level of reserves, the three regional banks decide whether to participate or not. Next, they offer a demand deposit contract to all agents willing to deposit their endowment in the bank

In the interim period, the interbank market allows banks with different needs for liquidity, to redistribute the reserves stored by them in the first period. The reserves are redistributed through cross deposits; the central bank coordinates this redistribution.

Our modeling of the interbank market is motivated by the way banks interconnect in reality and transfer securities from one to another. Furthermore, the existing electronic payment systems, typically controlled by central banks, play this role. As an example in Europe, consider the real time gross settlement system TARGET2.

The way we model depositors is standard in the bank runs literature. In the first period, agents deposit their endowment in their region's bank, which can be withdrawn in an interim or in a terminal period. There is a probability that a depositor is impatient, in which case she always withdraws in the interim period; patient depositors can choose in which period to withdraw. In the interim period, each depositor privately learns whether she is impatient or patient. Also, each depositor receives a noisy private signal about the quality of her bank's long-run investment, based on which she decides whether to withdraw or wait until the terminal period. Depositors in the three regions decide simultaneously.

Withdrawing the deposit in the terminal period has a higher expected return than early withdrawing, if the bank does not run out of resources in the interim period. A bank runs out of resources, if total withdrawals in the interim period are higher than the liquidation value of its total long-run investment. Since we assume that deposit contracts follow a first-come-first-serve rule and there is no deposit insurance, if the bank does not have enough resources in the interim period, there is a run. In this situation, early withdrawers may receive a payment, while late withdrawers receive nothing.

If the quality of banks' investment is observable, there are two equilibria, everyone runs and no one runs. If instead, the quality of banks' long-run investments is not observable and depositors receive noisy private signals about this quality, we show that equilibrium uniqueness is restored. This result is standard in global games, but this approach typically assumes that signals are infinitely precise, while we allow for some (but not any) imprecision in private signals. As in the standard result, we find that depositors in the three banks follow a trigger strategy, that is, they withdraw in the interim period if their signal is below a threshold.

We show that an interbank market, which redistributes reserves, and a central bank, which determines banks' reserve requirements and coordinates their redistribution, leads to a smaller probability of a bank run in all regions and to less inefficient bank runs, relative to the case with no interbank market and no central bank. The mechanism behind this crucial result works through depositors' beliefs, as follows.

The signal threshold for depositors is smaller if banks choose to participate in the interbank market. Intuitively, by redistributing reserves from liquid to illiquid banks, the interbank market provides banks with an insurance against the random interim demand of withdrawals. However, because of imperfect information about the quality of banks' investments, the insurance that the interbank market provides is incomplete. By adequately selecting the reserves to store and by coordinating the redistribution, the central bank improves that insurance. Depositors react to this double insurance by updating their propensity to run, which reinforces the former mechanism and leads to a smaller probability of a run and to less inefficient bank runs.

One implication of this result is that the interbank market's capacity to redistribute reserves and to reduce the *ex-ante* probability of a bank run is decreasing in the degree of correlation of the quality of banks' long run investments. In the extreme case, if banks' investments were of identical quality, depositors would receive similar private signals,[3] banks would have similar interim liquidity demand and there would be no reserves to redistribute.

We then study the way the central bank selects the fraction of reserves that banks must store, which in turn affects the interim payment that banks can afford to offer to depositors, contingent on participating in the interbank market.

We show that by adequately chosing the level of reserves to store, the central bank can improve the equilibrium outcome and enable banks to offer a higher interim payment to depositors, relative to the situation with no cross deposits. Since depositors are risk averse, a deposit contract offering a higher interim payment is *ex-ante* welfare improving to all agents.

We conclude that the constrained-efficient prudential regulation, in the form of reserve requirements, should not only consider the individual banks' transformation activities, but also the degree of correlation of risk exposures and the quality of information available. More specifically, when choosing the reserves to store, the central bank should take into account the pattern of correlation, the precision of private information and the term structure of interest rates. Failure to integrate these elements undermines financial stability and increases the *ex-ante* probability of bank run.

There is an immense literature on banks and bank runs. Although it cannot be fully covered here, in the following lines, we highlight two strands of this literature that are close to ours. See Gorton and Winton (2003) for a complete survey.

Our main contribution is that, by introducing imperfect information about the quality of banks' long-term investment, which can be correlated, we endogenously and uniquely determine the *ex-ante* probability of a bank run, when banks participate in an interbank market and the central bank determines the reserve requirements.

On the one hand, there is the literature that focuses on banks' liquidity provision and the

[3] Recall that we allow for some (but not any) imprecision in private signals.

way banks insure themselves through cross deposits. With a mechanism design perspective, Bhattacharya and Gale in 1986 show that banks, facing idiosyncratic liquidity shocks, have an incentive *ex-ante* to set up an interbank market. However, its setting up creates a free rider problem, which results in banks storing too few reserves and hence, a poor performance of the interbank market *ex-post*. We share with them the view of the central bank as a coordination device. We depart from them, because our goal is the endogenous determination of a unique probability of a bank run.

Allen and Gale in 1998 develop a model with an interbank market and a central bank, in which the interbank market is efficient and the probability of a bank run is endogenous. In line with them, our results show that the central bank can improve the equilibrium outcome. We depart from them, because a bank run in their model is never panic based, as it is here.

More recently, Freixas and Holthausen (2005), Freixas and Jorge (2008) and Heider, Hoerova and Holthausen (2008), among others, study the functioning of the interbank market, when it is not optimal. In particular, Freixas and Jorge study the micro-foundations of the monetary policy transmission mechanism, in the presence of interbank market imperfections. We share with them the conclusion that imperfect information about banks' liquidity shocks affects the functioning of the interbank market. We depart from them, because their aim is to study how financial imperfections affect monetary policy.

On the other hand, there is the literature on monetary regulation. Because banks' maturity transformation function exposes them to the risk of a bank run, this literature examines what is the best monetary regulation, *ex-ante* and *ex-post*, to minimize the probability of a bank run and importantly, the social costs associated to them. Fahri and Tirole (2011) and Freixas et al (2009) are two important contributions examining these issues.

We depart from this literature because our goal is not to characterize the optimal regulation, from a mechanism design perspective, but to study how an existing mechanism, that is, the mandatory reserve requirement determined by the central bank, affect depositors' propensity to rush and the *ex-ante* probability of a bank run, in the context of informational problems, correlation of investments and interbank market.

As an illustration, the reserve requirements considered here are, to some extent, closed to Kashyap, Rajan and Stein's (2008), who propose to mandate banks to store reserves, in the form of U.S. Treasury bonds, instead of cash as we do.

The paper proceeds as follows. Section two presents the model and discusses its main features. Section three studies the problem of depositors at the interim period. Section four analyses the problem of banks and that of the central bank, at the initial period. Concluding remarks are in section five. All proofs are relegated to the Appendix.

2 The economy

This section displays the setup of the model. Consider an economy with three periods, $t = 0, 1, 2$ and three regions, $j = A, B, C$. Each region is populated by a continuum of competitive banks, each with a continuum of agents. Agents are indexed by $i \in [0, 1]$ with total measure one. Also, there is a central bank, common to all regions.

Productive technology. At $t = 0$, agents and banks in each region can invest in a productive technology. The technology at each region yields a gross return $R > 1$ with probability $p(\widetilde{\theta}_j)$ or 0 with probability $1 - p(\widetilde{\theta}_j)$ at $t = 2$. If liquidated early, the gross return equals to one. The fundamental $\widetilde{\theta}_j$ is a random variable drawn at $t = 1$ but publicly observable at $t = 2$. The probability $p(.)$ is strictly increasing in θ_j, with $E_{\theta_j}[p(\theta_j)] \times R > 1$.

Alternatively, agents and banks in each region have access to a storage technology at $t = 0$, which yields a gross return of 1 at $t = 1, 2$.

Correlation of fundamentals across regions. At the beginning of $t = 1$, a random variable $\widetilde{\theta}$ is drawn from a uniform distribution with support $[0, 1]$. Also at $t = 1$, nature draws the fundamental state variable $h \equiv \{0, 1\}$: Independent or identical fundamentals, respectively. If $h = 1$, with probability q, fundamentals in the three regions are identical to the realized θ, namely, $\theta_A = \theta_B = \theta_C = \theta$. If $h = 0$, with probability $1 - q$, each $\widetilde{\theta}_j$ is independently drawn from another uniform distribution with support $[0, 1]$. θ and θ_j are only publicly observable at $t = 2$; h is publicly observed at $t = 1$. In brief,

$$\widetilde{\theta}_j = \begin{cases} \theta & \text{if } h = 1, \\ \sim U[0, 1] & \text{if } h = 0. \end{cases} \tag{1}$$

This specification allows us to model the correlation of fundamentals across regions, keeping the unconditional distributions identical for θ_A, θ_B and θ_C.

Agents. Agents consume a single good, which is divisible and storable. All agents are identical at $t = 0$ when they receive one unit of the consumption good as endowment. Agents' utility is

$$U(c_1 + \lambda_i c_2),$$

where $\{c_1, c_2\}$ denote consumption at $t = 1$ and $t = 2$. $U(\cdot)$ is continuous, twice differentiable, increasing and has a relative risk aversion coefficient greater than one.

At $t = 0$, agents can either invest in the productive technology or deposit their endowment in the bank of the region they live in.

At the beginning of $t = 1$, nature draws agents's type: impatient ($\lambda_i = 0$ with a probability μ) or patient ($\lambda_i = 1$ with probability $1 - \mu$). Agents' types are private information. Also at $t = 1$, agent i in region j observes an imperfect private signal

$$\widetilde{x}_{ij} = \theta_j + \widetilde{\varepsilon}_{ij} \tag{2}$$

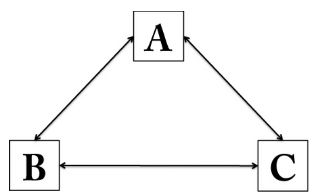
about the fundamental in her region θ_j, where the noise $\widetilde{\varepsilon}_{ij}$ has uniform distribution in $[-\varepsilon, \varepsilon]$ with $0 < \varepsilon < \frac{1}{4}$[4]. With this information, agent decides whether to liquidate her investment in the productive technology or withdraw her deposit from bank j at $t = 1$. Furthermore, assume that the expected utility from consuming at $t = 2$ is higher than from consuming at $t = 1$, that is,

$$E_{\theta_j}\left[p(\theta_j)\right] \times U(R) > U(1), \tag{3}$$

with $R > \frac{1}{\mu}$.[5]

Banks. Each region has a banking sector, with free entry. Since banks are *ex-ante* identical, make no profits and have access to the productive technology, they offer the same deposit contract that would be offered by a single regional bank that maximizes the welfare of agents. We denote the representative regional bank as A, B and C, respectively. Importantly, banks can cross deposit in a complete interbank market, as depicted in the next figure.

Figure 1. Complete interbank market structure.



At $t = 0$, bank j must first decide whether to participate in the interbank market at $t = 1$. If bank j chooses to participate, it must store at least $y \in [\mu, 1]$ as reserves, to be held in the central bank from $t = 0$ to $t = 1$, with y centrally determined, the same for all banks and publicly known. It invests the remaining $D_j - y$ in the productive technology, with $D_j \in [0, 1]$ total deposits received by bank j at $t = 0$. If instead bank j chooses not to participate, mandatory reserves are 0 bank j can invest all deposits. This choice is public information.[6]

At $t = 0$, bank j offers the following demand deposit contract to all agents willing to deposit their endowment in the bank. An agent demanding withdrawal at $t = 1$ is promised a fixed payment of $r_1 > 1$. If she waits until $t = 2$, she receives a stochastic return of r_{2j}, which is the net proceed of the productive investment, divided by the total withdrawals at $t = 2$, for $j = A, B, C$.

[4] To be explained later.

[5] $R > \frac{1}{\mu}$ introduces a lower bound on fundamentals.

[6] If choosing to participate, a bank will never store more than y from $t = 0$ to $t = 1$, because, first, the productive technology does as well as the storage technology if liquidating at $t = 1$ and even better in expected value if waiting until $t = 2$. Second, because there is no cost of early liquidation.

See forthcoming description of the central bank.

At $t = 1$, bank j must follow a sequential service constraint, that is, it pays r_1 until it runs out of resources and becomes bankrupt. Denoting n_j the proportion of agents in bank j withdrawing at $t = 1$, the deposit contract pays according to Table 1.[7]

Table 1: Deposit contract payments.

Withdrawal	If enough resources at $t = 1$	If not enough resources at $t = 1$
$t = 1$	r_1	r_1 with probability $\frac{D_j}{n_j r_1}$ [8] 0 with probability $1 - \frac{D_j}{n_j r_1}$
$t = 2$	r_{2j}	0

The central bank. Its role is to coordinate the storage of reserves. At $t = 0$, the central bank determines y, collects the reserves and invests them in the productive technology. At $t = 1$, under some to-be determined conditions, it helps re-allocating the reserves between banks.[9]

The interbank market. Its role is to redistribute at $t = 1$ the reserves stored by banks at $t = 0$.

We define $z_j^l \in R$ and $z_j^b \in R$ as the amount bank j lends and borrows in the interbank market at $t = 1$, respectively; $f_j \equiv z_j^b - z_j^l$ is the interbank net position. After observing h, n_j and $n_{j'}$, with $j' \neq j$, cross deposits settle at $t = 1$, according to table 2.

Table 2: Market clearing rules and cross deposit settlements at $t = 1$.

$t = 1$ Scenario		
$h = 0$		Cross deposits
No bankrupt and enough reserves	$n_j r_1 \leq y$	$z_j^l \in [0, y - n_j r_1]$ $z_j^b = 0$
No bankrupt but not enough reserves	$y < n_j r_1 < D_j$	$z_j^b \in [0, n_j r_1 - y]$ $z_j^l = 0$
Bankrupt	$n_j r_1 \geq D_j$	$z_j^l = z_j^b = 0$

If $h = 1$, regardless of n_j, $z_j^l = z_j^b = 0$.

[7] Banks' payoffs are to be determined, when describing the interbank market.

[8] The probability that an agent willing to withdraw at $t = 1$ receives the promised r_1 if the bank is bankrupt, π, solves:

$$n_j \times r_1 \times \pi = 1 \times D_j,$$

with $1 \times D_j$ the liquidation value of total deposits.

[9] As Battacharya and Gale (1986) have shown, although banks have an incentive *ex ante* to set up an interbank market, its setting up creates a free rider problem, which results in banks storing too few reserves and hence, a poor performance of the interbank market *ex post*. This motivates our modeling of the central bank, as a coordination device.

At $t = 1$, bank j can find itself in one of the three situations depicted in the previous table. Bank j is not bankrupt (it has enough resources at $t = 1$), whenever total $t = 1$ withdrawal $n_j r_1$ is smaller than the total liquidation value of deposits at $t = 1$, that is, $1 \times D_j$. Bank j is not bankrupt and has enough reserves, if it can pay r_1 to the proportion n_j willing to early withdraw by only using reserves y. Bank j is not bankrupt but it does not have enough reserves, if it can pay $n_j \times r_1$ by using not only y, but also cross deposits and/or partial liquidation. Finally, bank j is bankrupt at $t = 1$ if $n_j r_1$ is higher than $1 \times D_j$. If bankrupt, bank j fully liquidates the productive investment.

Whenever $y < n_j r_1 < D_j$, the way bank j serves $n_j r_1$ at $t = 1$ depends on the realized fundamental state variable, h. If $h = 1$, since $z_j^l = z_j^b = 0$, bank j needs to liquidate $n_j r_1 - y$. If instead $h = 0$ and reserves are not enough, bank j can borrow in the interbank market. Bank j's budget constraint at $t = 1$ if $h = 0$ becomes,

$$n_j r_1 = y + f_j. \tag{4}$$

At $t = 2$, if not bankrupt at $t = 1$, the non-liquidated productive investment pays off. Bank j pays the stochastic return r_{2j} to retail deposits withdrawn at $t = 2$ and a random payment to cross deposits. Importantly, cross deposits yield a gross interbank return R_f with probability $p(\theta_j)$ or 0 with probability $1 - p(\theta_j)$, with $1 < R_f < R$. Bank j's budget constraint at $t = 2$, provided not bankruptcy at $t = 1$, becomes,

$$(1 - n_j) r_{2j} = \begin{cases} (1 - y) \times R - f_j \times R_f & \text{with probability } p(\theta_j), \\ 0 & \text{with probability } 1 - p(\theta_j). \end{cases} \tag{5}$$

Timing. Figure 2: Order of events.

t=0				
Central bank selects y.	Bank j chooses whether to participate; it offers r_1 to depositors.	Depositor i in j decides whether to deposit or not.	Bank j collects deposits; it stores y or 0; it invests $1 - y$ or 1.	Central bank collects and invests y.

t=1			t=2	
θ_j is realized but not publ. observed.	Depositor i in j decides whether to withdraw or wait.	Cross deposits settle.	θ_j is publicly observed.	Investment, retail and cross deposits pay off.

3 The problem of depositors at $t = 1$

Section 3 treats the deposit contract as exogenous and y and r_1 as parameters. It solves for the equilibrium at $t = 1$, contingent on whether bank j chooses to participate in the interbank market at $t = 0$ or not. If bank j chooses not to cross deposit, the solution is identical to Goldstein and Pauzner (2005), except for the fact that we assume a non negative signal noise $\varepsilon > 0$, instead of $\varepsilon \to 0$.

3.1 No cross deposits

Let the subindex NC denote no cross deposits. We start with the conjectures that $r_1 \in \left[1, \frac{1}{\mu}\right]$ and that all agents deposit their endowment in the bank of the region they live in at $t = 0$, that is, $D_j = 1$. In addition, we conjecture that patient agents follow an unique monotone strategy, x^*_{NC}.

Definition 1. *A monotone strategy x^*_{NC} is such that a patient depositor with signal x_{ij} withdraws her deposit at t = 1 when $x_{ij} \leq x^*_{NC}$. Otherwise, she withdraws at t = 2.*

Denoting n_j as the total proportion of depositors who withdraw at $t = 1$ for each possible realization of θ_j, the $t = 2$ stochastic return, provided solvency at $t = 1$ is,

$$r^{NC}_{2j} = \begin{cases} \frac{1-n_j r_1}{1-n_j} R & \text{with probability } p(\theta_j), \\ 0 & \text{with probability } 1 - p(\theta_j). \end{cases} \tag{6}$$

with $f_j = 0$. The difference in utility of a patient agent from withdrawing at $t = 2$ rather than at $t = 1$ is,

$$v(\theta_j, n_j) = \begin{cases} p(\theta_j) U\left(\frac{1-n_j r_1}{1-n_j} R\right) - U(r_1) & \text{if } \mu \leq n_j < \frac{1}{r_1}, \\ 0 - \frac{1}{n_j r_1} U(r_1) & \text{if } \frac{1}{r_1} \leq n_j \leq 1. \end{cases} \tag{7}$$

If $\mu \leq n_j < \frac{1}{r_1}$ (first line in (7)), the proportion n_j of agents who early withdraw is lower than the maximum proportion the bank can serve at $t = 1$ without going bankrupt, $\frac{1}{r_1}$. A patient agent receives a random return if waiting (first term) or the promised r_1 if early withdrawing. Instead, if $\frac{1}{r_1} \leq n_j \leq 1$ (second line), the bank is bankrupt. A patient agent who waits until $t = 2$ receives nothing, while if withdrawing at $t = 1$ she may receive the promised r_1 with probability $\frac{1}{n r_1}$.

Let $n(\theta_j, x^*_{NC}(r_1))$ be the total proportion of withdrawals at $t = 1$ for each possible realization of θ_j. Given r_1, the process of signals x_{ij} in (2) and definition 1, $n(\theta_j, x^*_{NC}(r_1))$ is deterministic. It is given by,

$$n(\theta_j, x^*_{NC}(r_1)) = \mu + (1 - \mu) \Pr(x_{ij} < x^*_{NC}(r_1)).$$

When fundamentals θ_j are sufficiently bad, that is, below $x^*_{NC}(r_1) - \varepsilon$, all agents observe signals below $x^*_{NC}(r_1)$ and prefer to early withdraw. The proportion $n(\theta_j, x^*_{NC}(r_1))$ is thus equal to 1. Instead, when θ_j are above $x^*_{NC}(r_1) + \varepsilon$, all agents observe signals above $x^*_{NC}(r_1)$ and only impatient agents early withdraw, $n(\theta_j, x^*_{NC}(r_1) = \mu$. Because fundamentals and noise are uniformly distributed, $n(\theta_j, x^*_{NC}(r_1))$ decreases linearly between $x^*_{NC}(r_1) - \varepsilon$ and $x^*_{NC}(r_1) + \varepsilon$. We thus have

$$n(\theta_j, x^*_{NC}(r_1)) = \begin{cases} 1 & \text{if } \theta_j < x^*_{NC}(r_1) - \varepsilon, \\ \mu + (1-\mu)(\frac{1}{2} + \frac{x^*_{NC}(r_1) - \theta_j}{2\varepsilon}) & \text{if } \theta_j \in \left[x^*_{NC}(r_1) - \varepsilon, x^*_{NC}(r_1) + \varepsilon\right], \\ \mu & \theta_j > x^*_{NC}(r_1) + \varepsilon. \end{cases} \tag{8}$$

As in Goldstein and Pauzner (2005), we assume there are ranges of extremely good or extremely bad fundamentals, in which a patient agent's best action does not depend on her beliefs about other patient agents' behavior. They are defined by the lower and upper dominant bounds, $\underline{\theta}_{NC}(r_1)$ and $\overline{\theta}$, respectively.

When $\theta_j \in [0, \underline{\theta}_{NC}(r_1))$, the « lower dominance range », fundamentals are so low that the expected utility from waiting until $t = 2$ is lower than that from withdrawing at $t = 1$, even if all patient agents were to wait. Because the difference between x_{ij} and the true θ_j is no more than ε, a patient agent observing a signal $x_{ij} < \underline{\theta}_{NC}(r_1) - \varepsilon$ is sure that $\theta_j \in [0, \underline{\theta}_{NC}(r_1))$. Hence, she prefers to withdraw at $t = 1$, no matter her beliefs about what other agents do.

We compute $\underline{\theta}_{NC}(r_1)$ as the value of θ_j at which an agent is indifferent between r_1 and the random return if waiting, that is, $U(r_1) = p(\underline{\theta}_{NC})U\left(\frac{1-\mu r_1}{1-\mu}R\right)$. For any $r_1 \geq 1$, we assume there are feasible values of θ_j for which agents are sure that θ_j belongs to the lower dominance region. Since the support of θ_j is $[0, 1]$, $\underline{\theta}_{NC}(r_1)$ is increasing in r_1 and $r_1 \geq 1$, the condition that guarantees the former is that the lower bound at $r_1 = 1$ be $\underline{\theta}_{NC}(1) > 2\varepsilon$, or equivalently, $\underline{\theta}_{NC}(1) = p^{-1}\left(\frac{U(1)}{U(R)}\right) > 2\varepsilon$ at $r_1 = 1$.

For the upper dominance range, we follow Goldstein and Pauzner (2005), by modifying the gross return that the productive technology yields for an investment taken at $t = 0$ and liquidated at $t = 1$. Over the interval $(\overline{\theta}, 1]$, the technology now yields a certain gross return equal to R (instead of 1 in the range $[0, \overline{\theta}]$). Because over this interval, an agent who demands early withdrawal receives r_1, whereas an agent who waits until $t = 2$ receives $\frac{R - n_j r_1}{1 - n_j} > r_1$, all patient agents prefer to wait until $t = 2$. Since the support of θ_j is $[0, 1]$, we assume $\overline{\theta} < 1 - 2\varepsilon$.

An agent observing x_{ij} has an uniform posterior distribution about θ_j in $[x_{ij} - \varepsilon, x_{ij} + \varepsilon]$. Hence, definition 1, equations (8) and (7) and the posterior distribution of θ_j yield the expected utility differential of withdrawing at $t = 2$ instead of $t = 1$,

$$\triangle^{r_1}(x_{ij}, x^*_{NC}) = \frac{1}{2\varepsilon} \int_{x_{ij}-\varepsilon}^{x_{ij}+\varepsilon} v(\theta_j, n(\theta_j, x^*_{NC}))d\theta. \tag{9}$$

The equilibrium in monotone strategies is obtained by solving the indifference condition for the marginal patient depositor who withdraws at $t = 1$, that is,

$$\triangle^{r_1}(x^*_{NC}, x^*_{NC}) = 0. \tag{10}$$

Lemma 1. *There is an unique equilibrium in monotone strategies $x^*_{NC}(r_1)$ that solves (10). A patient depositor with signal x_{ij} withdraws at $t = 1$ if $x_{ij} \leq x^*_{NC}(r_1)$; otherwise, she withdraws at $t = 2$.*

Lemma 1 says that a patient agent's action is uniquely determined by her signal: She demands early withdrawal if and only if her signal is below the threshold. This result follows from the uniform distribution of fundamentals and noise in signals, θ_j and ε_{ij}, together with the existence of a lower and upper dominance range and $0 < \varepsilon < \frac{1}{4}$.[10]

If θ_j were observable ($\varepsilon = 0$), two equilibria would exist, as in Diamond and Dybvig (1983). One is a « no run » equilibrium, when only impatient agents withdraw at $t = 1$, so $n = \mu$, and all withdrawers receive r_1. The other one is a « run » equilibrium, when all impatient and patient agents withdraw at $t = 1$ and receive r_1 with probability $\frac{1}{r_1}$. As usual in global games, the introduction of signals with an arbitrary small noise allow to recover equilibrium uniqueness.

Importantly,

Lemma 2. *The signal threshold $x^*_{NC}(r_1)$ is increasing in r_1.*

When r_1 is larger, patient agents run in a larger set of signals. Hence, banks become more vulnerable to bank runs when they offer more risk sharing (higher r_1). The intuition is simple. If the $t = 1$ payment increases and the $t = 2$ payment decreases, the incentive of patient agents to early withdraw is also higher.

We define $\theta^*_{NC}(r_1)$ as the level of θ_j at which the total proportion of early withdrawals, $n(\theta^*_{NC}, x^*_{NC}(r_1))$, equals the maximum proportion bank j can serve at $t = 1$ without going bankrupt, $\frac{1}{r_1}$. Since θ_j is uniformly distributed in $[0, 1]$, $\theta^*_{NC}(r_1)$ is also the probability of a bank run,

$$\theta^*_{NC}(r_1) = x^*_{NC}(r_1) + \varepsilon \left(1 - 2\frac{1 - \mu r_1}{(1 - \mu) r_1}\right). \tag{11}$$

$\theta^*_{NC}(r_1)$ depends on the signal threshold $x^*_{NC}(r_1)$, the payment that the deposit contract prescribes for withdrawers at $t = 1$, r_1, and the boundaries of the support of the signal noise ε.

First, when the noise is collapsed to zero, $\varepsilon \to 0$, the probability of a bank run $\theta^*_{NC}(r_1)$ coincides with the threshold $x^*_{NC}(r_1)$. Second, $x^*_{NC}(r_1)$ does not have closed form solution unless the utility function and the process for $p(\theta_j)$ are specified. Third, whether the probability $\theta^*_{NC}(r_1)$ is smaller or higher than $x^*_{NC}(r_1)$ depends on the last term in (11), which goes from $-\varepsilon$ if $r_1 \to 1$ to ε if $r_1 \to \frac{1}{\mu}$. Last but not least, Lemma 2 and definition (11) imply that,

Lemma 3. *The probability $\theta^*_{NC}(r_1)$ is increasing in r_1.*

Bank j can only achieve more risk sharing, at the cost of a higher probability of a bank run. When r_1 increases, the expected payment of withdrawing at $t = 2$ decreases for any realization of θ_j, so incentives to run are higher. This incentive is further increased since, knowing that other agents are more likely to early withdraw, the agent assigns a higher probability to the event of bank run. The probability of a run then increases.

3.2 Cross deposits

Suppose now that banks choose to participate at $t = 0$ in the interbank market, with index CD. We make the same conjectures that in the previous section.

[10] The assumption $0 < \varepsilon < \frac{1}{4}$ guarantees that $0 < \underline{\theta} < \overline{\theta} < 1$.

The solution of the equilibrium at $t = 1$ with cross deposits follows exactly the same steps as in section 3.1. The monotone strategy x^*_{CD} is defined in accordance to definition 1, after replacing x^*_{NC} by x^*_{CD}. Provided no bankruptcy at $t = 1$, the $t = 2$ stochastic return r^{CD}_{2j} with cross deposits becomes,

$$r^{CD}_{2j} = \begin{cases} \frac{1-n_j r_1}{1-n_j} R & \text{with probability } p(\theta_j) \times q, \\ \frac{1-y}{1-n_j} R + \frac{y-n_j r_1}{1-n_j} R_f & \text{with probability } p(\theta_j) \times (1-q), \\ 0 & \text{with probability } 1 - p(\theta_j). \end{cases} \tag{12}$$

With probability q, fundamentals are identical (first line in (12)). Since the proportion n_j is the same in all regions, the interbank market is ineffective to redistribute reserves at $t = 1$. The random return if waiting until $t = 2$ equals the one a patient agent would receive if no cross deposits, with probability $p(\theta_j)$.[11] With probability $1 - q$, fundamentals are independent (second line). The random return if waiting until $t = 2$ is now the return of the productive investment (first term) net of payments to cross deposits (second term).[12] Finally, with probability $1 - p(\theta_j)$ (last line), the productive investment and the retail deposit if waiting pay off 0.

The functional form of the difference in utility for a patient depositor of withdrawing at $t = 2$ instead of $t = 1$ turns out to,

$$v(\theta_j, n_j) = \begin{cases} p(\theta_j) \left[qU\left(\frac{1-n_j r_1}{1-n_j} R\right) + (1-q)U\left(\frac{1-y}{1-n_j} R + \frac{y-n_j r_1}{1-n_j} R_f\right)\right] - U(r_1) & \text{if } \mu \leq n_j < \frac{1}{r_1}, \\ 0 - \frac{1}{n_j r_1} U(r_1) & \text{if } \frac{1}{r_1} \leq n_j \leq 1. \end{cases} \tag{13}$$

The only difference between (13) and (7) is the random return a patient agent receives if waiting until $t = 2$ if the bank is not bankrupt at $t = 1$. This return is now contingent on identical (with probability q) or independent fundamentals (probability $1 - q$) (first and second term in first line in (13), respectively). (13) and (7) share the contingent interim payment, which equals r_1 if the bank is not bankrupt or r_1 with probability $\frac{1}{n_j r_1}$ if bankrupt.

The total proportion of agents who withdraw at $t = 1$ from bank j at each possible realization of θ_j is still governed by equation (8),[13] while the expected utility differential of withdrawing at $t = 2$ instead of $t = 1$ is still governed by equation (9), after replacing (7) by (13) and x^*_{NC} by x^*_{CD}.

We need to modify the lower dominant bound with an interbank market, which now be-

[11] If $h = 1$ and $n_j r_1 < y$ (no bankrupt and enough reserves), bank j keeps the excess of liquidity $y - n_j r_1$ in its central bank account. At $t = 2$, total productive investment (including the excess of reserves) yields $(1 - y)R$ $+(y - n_j r_1)R = (1 - n_j r_1)R$ with probability $p(\theta_j)$ or 0 otherwise.

If $h = 1$ and $y < n_j r_1 < 1$ (no bankrupt but no enough reserves), bank j uses y and liquidates $n_j r_1 - y$ to serve $t = 1$ total withdrawals. At $t = 2$, total productive invesment yields $(1 - y)R-$ $(n_j r_1 - y)R = (1 - n_j r_1)R$, with probability $p(\theta_j)$ ($(n_j r_1 - y)R$ is the forgone interest).

[12] By plugging the $t = 1$ constraint (4) into the $t = 2$ budget constraint (5), the expression holds.

[13] Since the unconditional distributions for θ_A, θ_B and θ_C and the supports of their conditional distributions are identical and because of a common x^*_{CD}, agents share uniform beliefs about the proportion of agents who run for any θ_j. See the appendix.

comes $\underline{\theta}_{CD}(r_1)$. The value of θ_j at which a patient agent becomes indifferent with an interbank market, $\underline{\theta}_{CD}(r_1)$, is $U(r_1) = p(\underline{\theta}_{CD})\left(qU\left(\frac{1-\mu r_1}{1-\mu}R\right) + (1-q)U\left(\frac{1-y}{1-\mu}R + \frac{y-\mu r_1}{1-\mu}R_f\right)\right)$. For any $r_1 \geq 1$, the condition that guarantees that there are feasible values of θ_j for which agents are sure that θ_j belongs to the lower dominance region is $\underline{\theta}_{CD}(1) > 2\varepsilon$, or equivalently, $\underline{\theta}_{CD}(1) = p^{-1}\left(\frac{U(1)}{qU(R)+(1-q)U\left(\frac{1-y}{1-\mu}R+\frac{y-\mu}{1-\mu}R_f\right)}\right) > 2\varepsilon$.

The equilibrium monotone strategy x^*_{CD} satisfies,

$$\triangle^{r_1}(x^*_{CD}, x^*_{CD}) = 0. \tag{14}$$

Proposition 1. *There is an unique equilibrium in monotone strategies $x^*_{CD}(r_1)$ that solves (14). A patient depositor with signal x_{ij} withdraws at $t = 1$ if $x_{ij} \leq x^*_{CD}(r_1)$; otherwise, she withdraws at $t = 2$.*

Let $\underline{y} \equiv \mu r_1$ be the level of reserves, below which they are insufficient to serve the minimum possible $t = 1$ liquidity demand, μr_1. In the other extreme, let $\overline{y} \equiv \frac{1-\mu^2 r_1^2}{2(1-\mu r_1)}$ be the level of reserves above which the expected return if waiting until $t = 2$ with cross deposits is smaller than the expected $t = 2$ return without cross deposits.[14] The next corollary compares the two threshold strategies $x^*_{NC}(r_1)$ and $x^*_{CD}(r_1)$.

Proposition 2. *Assume $\underline{y} < y < \overline{y}$, $q < 1$ and $1 < R_f < r_1 < R$. The equilibrium in monotone strategies $x^*_{CD}(r_1)$ satisfies*

$$x^*_{CD}(r_1) < x^*_{NC}(r_1).$$

This is the main result of the section. Intuitively, because the interbank market allows banks to redistribute at $t = 1$ the reserves stored at $t = 0$ through cross deposits, the range of fundamentals over which bank j is bankrupt at $t = 1$ shrinks.[15] Hence, depositors update their beliefs and, provided the policy variables satisfy certain conditions, depositors reduce the signal threshold below which they withdraw at $t = 1$. In other words, cross deposits act as an insurance, against the liquidity risk.

The first condition of Proposition 2, $\underline{y} < y < \overline{y}$, guarantees that the interbank is effective to redistribute reserves at $t = 1$ and that depositors are at least indifferent between banks participating in the interbank market at $t = 0$ or not. The second condition, $1 < R_f < R$, entails the following liquidation pecking order, given $t = 1$ withdrawals: First, reserves, then cross deposits and finally, long-term investment. Also,

Corollary 1. *The signal threshold $x^*_{CD}(r_1)$ is increasing in r_1.*

Since θ_j continues to be uniformly distributed in $[0, 1]$, the probability of a run in bank j with interbank markets is

$$\theta^*_{CD}(r_1) = x^*_{CD}(r_1) + \varepsilon\left(1 - 2\frac{1-\mu r_1}{(1-\mu)r_1}\right), \tag{15}$$

[14] See the appendix for details.

[15] This is always true given the timing, the market clearing rules and assumptions on R_f and R.

with $\theta^*_{CD}(r_1)$ the level of θ_j below which bank j becomes bankrupt at $t = 1$, which is increasing in r_1. Proposition 2 and equation (15) imply that, under the same conditions, the bank run probabilities satisfy,

$$0 < \theta^*_{CD}(r_1) < \theta^*_{NC}(r_1) < 1. \tag{16}$$

The interbank market reduces the *ex-ante* probability of a bank run in all regions. The mechanism follows. Because by shuffling liquidity through the interbank market, cross deposits provide banks a buffer to serve the $t = 1$ demand of withdrawals, depositors update their beliefs and reduce their incentives to run, $x^*_{CD}(r_1)$, which in turn reinforces the former and leads to a smaller probability of a run.

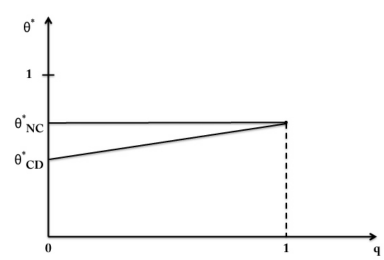
Importantly, the interbank market provides an insurance to banks against the random liquidity demand they face at $t = 1$. However, this insurance is incomplete. By determining the level of reserves to store at $t = 0$ and by coordinating their redistribution at $t = 1$, the central bank improves this insurance.

The extent to which $\theta^*_{NC}(r_1)$ and $\theta^*_{CD}(r_1)$ differ depends on how noisy the signals that depositors receive about the quality of their bank' investment are and how probable it is that the quality of banks' investments is identical. The noise of signals is governed by the parameter ε, while the probability of identical quality is governed by q. The next corollary studies the link between the probabilities $\theta^*_{CD}(r_1)$, $\theta^*_{NC}(r_1)$ and q.

Corollary 2. *Assume $\underline{y} < y < \overline{y}$ and $1 < R_f < r_1 < R$. If $q = 1$, $\theta^*_{CD}(r_1) = \theta^*_{NC}(r_1)$. Otherwise, $\theta^*_{CD}(r_1)$ is increasing in q.*

Interestingly, as illustrated in figure 3, the interbank market's capacity to redistribute reserves and to reduce the *ex-ante* probability of a run is decreasing in q. This is because when fundamentals are identical, which occurs with probability q, banks have the same $t = 1$ liquidity demand and there are no reserves to redistribute.

Figure 3: Probabilities of a run $\theta^*_{CD}(r_1)$ and $\theta^*_{NC}(r_1)$, as function of q.



To conclude, we compare the *ex-ante* probabilities of a bank run, with and without cross deposits, with the lower bound on fundamentals.

We define the size of the « panic based inefficiency » as the range of fundamentals where inefficient panic based bank runs occur, above the lower bound on fundamentals. Note that only when fundamentals θ_j are below the lower bound, $\underline{\theta}_{CD}(r_1)$ or $\underline{\theta}_{NC}(r_1)$, it is efficient for the bank to early liquidate the long term investment. The size of the panic based inefficiency equals $|\theta^*_{CD}(r_1) - \underline{\theta}_{CD}(r_1)|$ if banks choose to participate in the interbank market, while it equals $|\theta^*_{NC}(r_1) - \underline{\theta}_{NC}(r_1)|$ if choosing not to participate. Interestingly,

$$|\theta^*_{CD}(r_1) - \underline{\theta}_{CD}(r_1)| < |\theta^*_{NC}(r_1) - \underline{\theta}_{NC}(r_1)|.^{16} \tag{17}$$

The range of fundamentals of inefficient panic based bank runs shrinks with an interbank market. Intuitively, the interbank market allows banks to insure themselves against the random liquidity demand they face at $t = 1$. The central bank improves this insurance, by coordinating the storage of reserves at $t = 0$ and their redistribution at $t = 1$. The combination of these two effects reduces the panic based inefficiency. Importantly, provided the economy parameters satisfy certain conditions, public intervention improves the equilibrium outcome.

4 Demand deposit contract and monetary regulation

Section 4 lets the deposit contract offered by banks A, B and C to depositors at $t = 0$ and the policy variable y to be endogenous.

First, for benchmarking, Section 4 briefly revisits results from Diamond and Dibvig (1983) regarding the equilibrium outcome under autarky and first-best contracts. Second, it examines the way banks determine the equilibrium interim payment to offer to depositors, contingent on choosing not to or to cross deposit at $t = 0$. Recall that if bank j chooses not to cross deposit, the solution is identical to Goldstein and Pauzner (2005). Third, it studies banks' optimal choice between storage and investment *ex-ante*, which is summarized in their choice to participate in the interbank market. Finally, it studies how the selection of y influences banks' and depositors' decisions.

Banks and central bank Banks face two types of risk. On the one hand, banks face a liquidity risk, defined as uncertainty about $t = 1$ liquidity demand, due to private information on agents' types. On the other hand, they are exposed to aggregate uncertainty, due to private information about θ_j.

The central bank has two tools to help banks manage these risks and avoid bank runs.[17] The first tool is *ex-ante* prudential regulation, as given by the selection of y. The second tool is the coordination of the liquidity redistribution, *ex-post*.

The central bank selects the socially optimal level of y to maximize the *ex-ante* expected utility of a representative depositor. Importantly, because authority and banks have the same information

[16] See the appendix for a derivation.

[17] In reality, central authorities have a much richer menu of tools. See Kashyap, Stein and Rajan (2008) for a complete survey and Fahri and Tirole (2011) for a careful analysis.

and have access to the same productive technology at $t = 0$, banks are *ex-ante* identical, they make no profits and there are no externalities among banks,[18] the central bank pins down y and relies on banks to determine at $t = 0$, the equilibrium interim payment and to reveal at $t = 1$, their interim liquidity demand.

4.1 Autarky and First Best Solution

Since agents are risk averse, a transfer of consumption from patient to impatient agents is *ex-ante* beneficial to all agents. If types were observable, it would be possible to implement a contract that optimally shares risk between patient and impatient agents.

Lemma 4. *(Optimal risk sharing) The first best allocation in the economy would be as follows. At $t = 0$, all agents invest in the productive technology. At $t = 1$, impatient agents receive $c_1^{FB} > 1$ and consume it. Patient agents receive nothing. At $t = 2$, patient agents receive $c_2^{FB} = \frac{1-\mu c_1^{FB}}{1-\mu} \times R < R$ with probability $p(\theta_j)$ and 0 with probability $1 - p(\theta_j)$.*

When agents' types are not public information, this insurance contract contingent on types is not possible. In what follows, we study how banks can increase welfare relative to the autarkic situation.

4.2 The problem of banks at $t = 0$

Let the contingent interim payment offered by bank j to depositors, contingent on choosing not to or to cross deposit at $t = 0$ be r_1^{NC} and r_1^{CD}, respectively.

4.2.1 No cross deposits

Bank j determines r_1^{NC} so as to maximize the (to be defined) *ex-ante* conditional expected utility of a representative depositor who is promised r_1^{NC} if early withdrawing. The conditional expected utility depends on the payments under all possible values of θ_j. Since $\theta_j \in [0, 1]$ is unknown at $t = 0$, there are four intervals of θ_j to consider, as depicted in the next table.

Figure 4: Run and no run and the proportion of $n(\theta_j, x^*_{NC}(r_1^{NC}))$, as a function of θ_j. No cross deposits.

θ_j	$\left[0, \theta^*_{NC}(r_1^{NC})\right]$	$\left[\theta^*_{NC}(r_1^{NC}), x^*_{NC}(r_1^{NC}) + \varepsilon\right)$	$\left(x^*_{NC}(r_1^{NC}) + \varepsilon, \overline{\theta}\right]$	$\left[\overline{\theta}, 1\right]$
$n(\theta_j, x^*_{NC}(r_1^{NC}))$	1	$\in [\mu, \frac{1}{r_1^{NC}})$	μ	μ
Outcome in j	Run	No Run	No Run	No Run

[18] Because agents can only deposit in the bank of the region they live in, the contract that one bank can offer to its depositors does not affect the payoffs of an agent who deposits in another bank.

When $\theta_j \in [0, \theta^*_{NC}(r_1^{NC})]$ (first interval), the bank liquidates all its investments at $t = 1$ and depositors of both types receive r_1^{NC} with probability $\frac{1}{r_1^{NC}}$. When $\theta_j \in (\theta^*_{NC}(r_1^{NC}),\ x^*_{NC}(r_1^{NC}) + \varepsilon]$ (second interval), the proportion $n(\theta_j, x^*_{NC}(r_1^{NC}))$ of depositors who withdraw at $t = 1$ receive r_1^{NC}, while $1 - n(\theta_j, x^*_{NC}(r_1^{NC}))$ of patient depositors who wait until $t = 2$ receive $\frac{1-n_j r_1^{NC}}{1-n_j} R$ with probability $p(\theta_j)$ or 0 otherwise. When $\theta_j \in (x^*_{NC}(r_1^{NC}) + \varepsilon,\ \overline{\theta}]$ (third interval), all patient depositors wait until $t = 2$ and receive $\frac{1-\mu r_1^{NC}}{1-\mu} R$ with probability $p(\theta_j)$. Finally, when $\theta_j \in (\ \overline{\theta}, 1]$ (last one), all patient depositors waiting until $t = 2$ always receive $\frac{R-\mu r_1^{NC}}{1-\mu}$.

The conditional expected utility of a representative depositor who is promised r_1^{NC}, conditional on bank j choosing not to cross deposit is

$$\Omega^{NC} = \max_{r_1^{NC}} \left\{ \begin{array}{c} \int_0^{\theta^*_{NC}(r_1^{NC})} \frac{1}{r_1^{NC}} U(r_1^{NC}) d\theta_j \\ + \int_{\theta^*_{NC}(r_1^{NC})}^{x^*_{NC}(r_1^{NC})+\varepsilon} \left(n_j U(r_1^{NC}) + (1-n_j) p(\theta_j) U\left(\frac{1-n_j r_1^{NC}}{1-n_j} R \right) \right) d\theta_j \\ + \int_{x^*_{NC}(r_1^{NC})+\varepsilon}^{\overline{\theta}} \left(\mu U(r_1^{NC}) + (1-\mu) p(\theta_j) U\left(\frac{1-\mu r_1^{NC}}{1-\mu} R \right) \right) d\theta_j \\ + \int_{\overline{\theta}}^{1} \left(\mu U(r_1^{NC}) + (1-\mu) \left(U\left(\frac{R-\mu r_1^{NC}}{1-\mu} \right) \right) \right) d\theta_j \end{array} \right\}, \qquad (18)$$

with each line corresponding to one interval in Figure 4 and n_j abbreviating for $n(\theta_j, x^*_{NC}(r_1^{NC}))$.

The next proposition shows that, provided the lower dominance range $\theta_j \in [0, \underline{\theta}_{NC}(1)]$ is not too large, the demand deposit contract offered by bank j, which is summarized in r_1^{NC}, satisfies,

Lemma 5. *If choosing not to cross deposit, the equilibrium interim payment that bank j offers to depositors, r_1^{NC}, satisfies*

$$1 < r_1^{NC} < c_1^{FB}.$$

The demand deposit contract is socially desirable, even when the cost of bank run is considered. Because of the high coefficient of risk aversion, the deposit contract with $r_1^{NC} > 1$ achieves higher welfare than that reached under autarky. However, the interim payment r_1^{NC} is still inferior to the first best allocation, c_1^{FB}. This is because bank j needs to consider the effect that an increase of r_1^{NC} has on the probability of a run, when agents' types are not known.

Increasing r_1^{NC} above 1 has some benefits, but also some costs. It is beneficial since it enables risk sharing among agents. However, it is also costly because of two effects. First, it widens the range in which bank runs occur and the investment is liquidated slightly beyond $\underline{\theta}_{NC}(1)$. Second, in the range $[0, \underline{\theta}_{NC}(1))$, it makes runs more costly. This is because setting r_1^{NC} above 1 causes some agents not to get any payment (due to the sequential service constraint). Provided the range $[0, \underline{\theta}_{NC}(1)]$ is not too large, the net effect is positive and the deposit contract is welfare improving.

Importantly, inefficient, panic based bank runs occur in equilibrium. This is seen by noting that $\theta^*_{NC}(r_1^{NC})$ is larger than $\underline{\theta}_{NC}(r_1^{NC})$ whenever r_1^{NC} is above 1.

4.2.2 Cross deposits

The decision to cross deposit does not affect the way banks determine the interim payment offered to depositors. As illustrated in Figure 5, the program to solve with cross deposits shares with (18) the four intervals of θ_j to consider and the proportion of agents who withdraw at $t = 1$ in each interval, which now equals $n(\theta_j, x^*_{CD}(r_1))$.

Figure 5: Run and no run as a function of θ_j, with cross deposits.

θ_j	$[0, \theta^*_{NC}(r_1^{NC})]$	$[\theta^*_{NC}(r_1^{NC}), x^*_{NC}(r_1^{NC}) + \varepsilon)$	$(x^*_{NC}(r_1^{NC}) + \varepsilon, \overline{\theta}]$	$[\overline{\theta}, 1]$
$n(\theta_j, x^*_{NC}(r_1^{NC}))$	1	$\in [\mu, \frac{1}{r_1^{NC}})$	μ	μ
Outcome in j	Run	No Run	No Run	No Run

The only difference with program (18) is the random return if waiting until $t = 2$, which is now contingent on whether fundamentals are identical, with probability q, or independent, with probability $1 - q$. Consider the possible values that the $t = 2$ random return can take. When $\theta_j \in (\ \theta^*_{CD}(r_1^{CD}),\ x^*_{CD}(r_1^{CD}) + \varepsilon]$, the proportion $1 - n(\theta_j, x^*_{CD}(r_1^{CD}))$ of patient depositors who wait until $t = 2$ receive $\frac{1-n_j r_1^{CD}}{1-n_j} R$ with probability $p(\theta_j) \times q$ or $\frac{1-y}{1-n_j} R + \frac{y-n_j r_1^{CD}}{1-n_j} R_f$ with probability $p(\theta_j) \times (1-q)$ or 0 otherwise. When $\theta_j \in (x^*_{CD}(r_1^{CD}) + \varepsilon,\ \overline{\theta}]$, all patient depositors wait until $t = 2$ and receive $\frac{1-\mu r_1^{CD}}{1-\mu} R$ with probability $p(\theta_j) \times q$ or $\frac{1-y}{1-\mu} R + \frac{y-\mu r_1^{CD}}{1-\mu} R_f$ with probability $p(\theta_j) \times (1-q)$ or 0 otherwise. Finally, when $\theta_j \in (\overline{\theta}, 1]$, all patient depositors always receive a positive $t = 2$ return, which equals $\frac{R-\mu r_1^{CD}}{1-\mu}$ with probability q or $\frac{R-y}{1-\mu} + \frac{y-\mu r_1^{CD}}{1-\mu} R_f$ with probability $(1 - q)$.

The *ex-ante* conditional expected utility of a representative depositor in bank j with cross deposits, Ω^{CD}, is now

$$\Omega^{CD} = \max_{r_1^{CD}} \left\{ \begin{array}{c} \int_0^{\theta^*_{CD}(r_1^{CD})} \frac{1}{r_1^{CD}} U(r_1^{CD}) d\theta_j \\ + \int_{\theta^*_{CD}(r_1^{CD})}^{x^*_{CD}(r_1^{CD})+\varepsilon} \left(n_j U(r_1^{CD}) + (1-n_j) p(\theta_j) \left(qU\left(\frac{1-n_j r_1^{CD}}{1-n_j} R \right) + (1-q) U\left(\frac{1-y}{1-n_j} R + \frac{y-n_j r_1^{CD}}{1-n_j} R_f \right) \right) \right) d\theta_j \\ + \int_{x^*_{CD}(r_1^{CD})+\varepsilon}^{\overline{\theta}} \left(\mu U(r_1^{CD}) + (1-\mu) p(\theta_j) \left(qU\left(\frac{1-\mu r_1^{CD}}{1-\mu} R \right) + (1-q) U\left(\frac{1-y}{1-\mu} R + \frac{y-\mu r_1^{CD}}{1-\mu} R_f \right) \right) \right) d\theta_j \\ + \int_{\overline{\theta}}^{1} \left(\mu U(r_1^{CD}) + (1-\mu) \left(qU\left(\frac{R-\mu r_1^{CD}}{1-\mu} \right) + (1-q) U\left(\frac{R-y}{1-\mu} + \frac{y-\mu r_1^{CD}}{1-\mu} R_f \right) \right) \right) d\theta_j \end{array} \right\} \quad (19)$$

If choosing to cross deposit, the interim payment that bank j offers to depositors at $t = 0$ depends on y.

Proposition 3. *Provided $\underline{\theta}_{CD}(1)$ is not too large and $\underline{y} < y < \overline{y}$, the equilibrium interim payment that bank j offers to depositors if choosing to cross deposit, r_1^{CD}, satisfies,*

$$1 < r_1^{CD} < c_1^{FB}.$$

Proposition 3 shows that the demand deposit contract with cross deposits achieves higher welfare than that reached under autarky, but it is still inferior to the first best allocation, as it generates inefficient, panic based bank runs. As before, this is because $\theta^*_{CD}(r_1^{CD})$ is larger than the lower bound $\underline{\theta}_{CD}(r_1^{CD})$, whenever r_1^{CD} is above 1. Importantly, the market failure driving the inefficiency $\theta^*_{CD}(r_1^{CD}) > \underline{\theta}_{CD}(r_1^{CD})$ is due to imperfect information, because neither agents' types nor the quality of banks' long-term investment are observed at $t = 0$.

Knowing that both demand deposit contracts achieve higher welfare than that reached under autarky, two questions arise. First, when do banks choose to participate in the interbank market? Second, how does the equilibrium interim payment with cross deposits compare to the interim payment without cross deposits, that is, is r_1^{CD} higher than r_1^{NC}? The next section answers these questions.

4.2.3 Decision to participate and interim payment

First, we analyze the bank decision to participate in the interbank market.

Whether bank j chooses to participate in the interbank market at $t = 0$ depends on the reserves to store, y. This is because the level of y modifies the marginal cost of better risk sharing, as measured by $\theta^*_{CD}(r_1^{CD})$.

Lemma 6. *Bank j chooses to participate in the interbank market at $t = 0$ if reserves are such that $\theta^*_{CD}(r_1^{CD}) \leq \theta^*_{NC}(r_1^{NC})$. Otherwise, bank j chooses not to participate.*

Each bank trades off the insurance that cross deposits provide against the random interim demand of withdrawals, with the foregone long-term return that a bank looses, when storing a fraction of its deposits as reserves. Whenever this insurance leads to a lower probability of bank run, relative to the situation with no cross deposits, the bank prefers to participate in the interbank market.

Second, we compare interim payments r_1^{CD} and r_1^{NC}.

Proposition 4. *Assume $\underline{y} < y < \overline{y}$, $1 < R_f < r_1^{NC} < R$ and $1 < R_f < r_1^{CD} < R$. The equilibrium interim payment that bank j offers to depositors, with or without cross deposits, r_1^{CD} and r_1^{NC}, respectively, satisfy*

$$c_1^{FB} > r_1^{CD} \geq r_1^{NC} > 1.$$

Whenever by redistributing reserves, the interbank market reduces the *ex-ante* probability of a run and thus, the marginal cost of better risk sharing, bank j can afford a higher interim payment to depositors, resulting in $r_1^{CD} \geq r_1^{NC} > 1$.

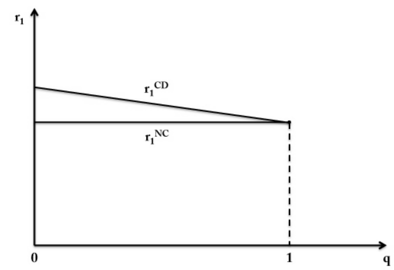
More generally, because of the nature of the problem, with agents trading with banks and banks trading among themselves in the presence of imperfect information, *ex-ante* constrained-efficiency could only be restored if an insurance mechanism to agents or banks would exist. This is precisely the role of the interbank market, complemented by the central bank: Banks insure themselves through cross deposits and the central bank improves the insurance. As a result, the equilibrium interim payment r_1^{CD} is closer to c_1^{FB} (relative to the situation with no interbank market and no central bank). Public intervention improves the equilibrium outcome.

To conclude, we study how the interim payment changes with the probability q.

Corollary 3. *Assume* $\underline{y} < y < \overline{y}$, $1 < R_f < r_1^{NC} < R$ *and* $1 < R_f < r_1^{CD} < R$. *If* $q = 1$, $r_1^{CD} = r_1^{NC}$. *Otherwise,* r_1^{CD} *is decreasing in* q.

Intuitively, if $q = 1$, fundamentals are identical and the interbank market is ineffective to redistribute reserves. The interim payment that bank j could offer would then equal the interim return with no cross deposits, that is, $r_1^{CD} = r_1^{NC}$. Instead, if $q < 1$, the interbank market can redistribute reserves. However, its capacity is decreasing in q, since the probability of a bank run is increasing in q. As illustrated in Figure 6, both elements combined imply that the interim payment that bank j can afford with cross deposits is also decreasing in q.

Figure 6: Interim payments r_1^{NC} and r_1^{CD}, as function of q.



4.3 The problem of the central bank at $t = 0$

We study now the central bank's *ex-ante* prudential regulation, that is, how it pins down the level y of reserves to store at $t = 0$.

The central bank selects y so as to maximize the *ex-ante*, unconditional expected utility of representative depositor in bank j, who is promised r_1^{CD} if early withdrawing, subject to

$$\underline{y} < y < \overline{y}. \tag{20}$$

The problem to solve simplifies when taking into account the following elements. To start with, the central bank faces no commitment problem or moral hazard problem about banks. Second, banks are *ex-ante* identical, they determine the deposit contract simultaneously at $t = 0$ and, at $t = 1$, they have no incentive to misreport their interim demand of withdrawal.[19] Third, regardless of its decision to cross deposit or not, bank j already determines the equilibrium interim payment, so as to maximize the expected utility of a representative depositor. In particular, as reflected in program (19), the equilibrium r_1^{CD} depends on y. Finally, because the interbank market is socially desirable, the central bank should select y, so as to encourage banks to prefer to participate in the interbank market at $t = 0$.

[19] By reporting n_j and because of equation (8), θ_j becomes uniquely determined at $t = 1$.

Assuming the economy parameters are such that an interior solution exists, the central bank pins down y, such that,

$$\theta^*_{NC}(r_1^{NC}) = \theta^*_{CD}\left(r_1^{CD}\right). \tag{21}$$

The central bank selects y so as to allow banks choosing to cross deposit, to offer the highest possible interim payment to depositors at $t = 0$, given the *ex-ante* probability of a run with no cross deposits, $\theta^*_{NC}(r_1^{NC})$.

We now explain why the central bank proceeds this way. First, recall that a bank chooses to participate in the interbank market only if the probability of a bank run with cross deposits is not greater than the likelihood of a bank run with no cross deposits. Second, because the probability of a bank run is increasing in the interim payment, the likelihood of a bank run with no cross deposits constrains the selection of reserves y and hence, the maximum interim payment r_1^{CD} to offer. Finally, because agents are risk averse, this way of selecting the reserve requirement is *ex-ante* welfare improving to all agents. Crucially, choosing y in this way implies that,

Corollary 4. *The equilibrium interim payment that bank j offers to depositors with cross deposits, r_1^{CD}, satisfy*

$$c_1^{FB} > r_1^{CD} > r_1^{NC} > 1.$$

The central bank intervention improves the insurance that banks provide themselves through cross deposits, leading to a lower probability of a bank run, and importantly, allowing banks to offer a higher interim payment, relative to the situation with no cross deposits, $r_1^{CD} > r_1^{NC}$. Because of the high coefficient of risk aversion, the deposit contract with $r_1^{CD} > r_1^{NC} > 1$ improves welfare with respect to autarky and relative to the situation with no interbank market.

More generally, while prudential supervision is traditionally concerned with the solvency of individual institutions, Corollary 4 stresses that *ex-ante* constrained-efficient prudential regulation, in the form of reserve requirements, should not only consider the individual banks' transformation activities, but also the degree of correlation of risk exposures and the quality of information available. Importantly, failure to integrate these elements undermines financial stability and increases the *ex-ante* probability of bank run.

5 Conclusion

This paper provides micro foundations for the interbank market role in allocating liquidity, which is important to better understand how central banks should respond to liquidity shocks.

We show that the coexistence of an interbank market, which redistributes reserves, and a central bank, which determines the reserve requirements and coordinates their redistribution, leads to a smaller probability of a bank run in all regions and to less inefficient bank runs, relative to the case with no interbank market. By adequately choosing the level of reserves to store, the central bank can improve the equilibrium outcome.

We now discuss how to apply and extend our results.

One possible application is public policy analysis. Our model allows us to endogenously determine the probability of bank run, when the quality of banks' investments is correlated and banks participate in an interbank market. This probability is key to assess the desirability of public policies that attempt to avoid runs.

For example, during the 2007-2008 banking crisis, several episodes of bank runs (such as Northern Rock in the United Kingdom, in 2007 or IndyMac in the United States, in 2008) became sadly famous. With the goal of promoting a more resilient banking sector, the Basel Committee on Banking Supervision and the G20 launch in 2010-2011 a new banking regulation, known as Basel III. Basel III introduces changes in capital requirements, together with new liquidity requirements (the net stable funding ratio and the liquidity coverage ratio) in order to strengthen global capital and liquidity regulations.

It could then be possible to apply our model to assess whether such policy measures are desirable or specify under which conditions they can be. Indeed, our reserve requirements could, to some extent, be interpreted as a simplified Basel III liquidity coverage ratio.

The first venue of future research is to study how the introduction of a central bank facing a moral hazard problem about banks affect banking panics.[20] More precisely, the aim is to study how this feature alters the way the central bank determines the reserves requirements, the way banks determine the equilibrium interim payment they offer to depositors and the probability of a bank run.

The second venue is to increase the size of the interbank market to include a finite number of banks, higher than three. The goal is to generalize our results by allowing for different architectures of interbank market.

[20] Moral hazard refers to the tendency of an insured (the bank) to relax its effort to prevent the occurence of the risk that it has been insured against, once it has shifted the risk to the insurance institution. In our context, once depositors have deposited their cash, the bank may have incentives to take more risks and thereby, expropiate value to depositors.

6 Appendix. Proofs

6.1 Definition of $n(\theta_j, x^*_{NC}(r_1))$ and $n(\theta_j, x^*_{CD}(r_1))$

To compute the proportion $n(\theta_j, x^*_{NC}(r_1))$ of agents who run early at each θ_j, conditional on sharing the same threshold $x^*_{NC}(r_1)$, we split the interval of x_{ij} in three subintervals and we compute this proportion for each of them.

Subinterval 1: $x_{ij} > x^*_{NC}(r_1) + \varepsilon$

This situation is only possible if $\theta_j > x^*_{NC}(r_1)$. Thus, no patient agent should demand early withdrawal.

Subinterval 2: $x_{ij} < x^*_{NC}(r_1) - \varepsilon$

In this range, fundamentals are so low that all patient agent should early withdraw (because it is always true that $\theta_j < x^*_{NC}(r_1)$)

Subinterval 3: $x_{ij} \in [x^*_{NC}(r_1) - \varepsilon, x^*_{NC}(r_1) + \varepsilon]$

This is the most interesting case. First, note that given θ_j and ε, the signal x_{ij} is uniformly distributed over $[\theta_j - \varepsilon, \theta_j + \varepsilon]$.

Second, according to definition 1, a patient agent runs when she observes a signal $x_{ij} \leq x^*_{NC}(r_1)$. Given r_1, ε and the threshold strategy $x^*_{NC}(r_1)$, the proportion of agents early withdrawing at state θ_j, after observing the signal x_{ij}, is:

$$\begin{aligned} n(\theta_j, x^*_{NC}(r_1)) &= \mu + (1-\mu)\int_{\theta_j-\varepsilon}^{x^*_{NC}(r_1)} \frac{1}{2\varepsilon} dx_{ij} \\ n(\theta_j, x^*_{NC}(r_1)) &= \mu + (1-\mu)(\frac{1}{2} + \frac{x^*_{NC}(r_1) - \theta_j}{2\varepsilon}) \end{aligned}$$

Adding up, the proportion of agents that run at each θ_j takes the following form:

$$n(\theta_j, x^*_{NC}(r_1)) = \begin{cases} 1 & \text{if } \theta_j < x^*_{NC}(r_1) - \varepsilon, \\ \mu + (1-\mu)(\frac{1}{2} + \frac{x^*_{NC}(r_1)-\theta_j}{2\varepsilon}) & \text{if } x^*_{NC}(r_1) - \varepsilon \leq \theta_j \leq x^*_{NC}(r_1) + \varepsilon, \\ \mu & \text{if } \theta_j > x^*_{NC}(r_1) + \varepsilon. \end{cases} \tag{22}$$

Concerning $n(\theta_j, x^*_{CD}(r_1))$, since (a) the unconditional distributions for θ_A, θ_B and θ_C and the supports of their conditional distributions are identical; (b) agents share the same threshold $x^*_{CD}(r_1)$; (c) there are no externalities among banks and (d) the market clearing rules for the interbank market, as defined in table 2.1, it is still true that with cross deposits,

Lemma 7. *Agents share uniform beliefs about the proportion of agents who run at each θ_j, which becomes,*

$$n(\theta_j, x^*_{CD}(r_1)) = \begin{cases} 1 & \textit{if } \theta_j < x^*_{CD}(r_1) - \varepsilon, \\ \mu + (1-\mu)(\frac{1}{2} + \frac{x^*_{CD}(r_1)-\theta_j}{2\varepsilon}) & \textit{if } x^*_{CD}(r_1) - \varepsilon \leq \theta_j \leq x^*_{CD}(r_1) + \varepsilon, \\ \mu & \textit{if } \theta_j > x^*_{CD}(r_1) + \varepsilon. \end{cases} \tag{23}$$

6.2 Proof of Lemma 1

6.2.1 Existence

We restrict attention to threshold strategies, according to definition 1. We prove that there exists at least one threshold strategy x^*_{NC} such that $\Delta^{r_1}(x^*_{NC}, x^*_{NC}) = 0$:

At $x_{ij} = x^*_{NC}(r_1)$, the marginal depositor is indifferent between withdrawing at $t = 2$ or $t = 1$. Consider the following elements. First, $\Delta^{r_1}(x_{ij}, x^*_{NC})$ is a real-valued continuous function on the interval $[a, b]$, with $[a, b]$ such that $a \leq x^*_{NC}(r_1) \leq b$. Second, assuming that $a \in [0, \underline{\theta}_{NC}(1))$ and $b \in (\overline{\theta}, 1]$, the relevant interval is $[a, b] \subseteq [0, 1]$. Third, suppose there exists a number u such that $u \in [\Delta^{r_1}(a, x^*_{NC}), \Delta^{r_1}(b, x^*_{NC})]$. Fourth, by the existence of lower and upper dominance range, $\Delta^{r_1}(a, x^*_{NC}) < 0$ for any $a \in [0, \underline{\theta}_{NC}(1))$ and $\Delta^{r_1}(b, x^*_{NC}) > 0$ for any $b \in (\overline{\theta}, 1]$. Without any loss in generality, we consider $u = 0$.

Hence, by the intermediate value theorem, there exists at least one threshold equilibrium $x^*_{NC}(r_1) \in [0, 1]$, such that:

$$\Delta^{r_1}(x^*_{NC}, x^*_{NC}) = 0$$

Moreover, given the single crossing condition of the function $v(\cdot)$, $x^*_{NC}(r_1)$ is exactly the unique threshold equilibrium .

The second step requires to prove that any equilibrium must be a threshold equilibrium. In the next subsection, we provide some intuition of the proof. See Goldstein and Pauzner (2005) for a formal proof. It rests on the assumption of the uniform distribution for the fundamentals and the error term.

6.2.2 Uniqueness

The usual argument that shows that with noisy signals, there is a unique equilibrium (see Carlson van Damme (1993) and Morris and Shin (1998)) builds on the property of global strategic complementaries or action monotonicity of the payoff function: An agent's incentive to take an action is higher when more other agents take that action. Mathematically, it requires that the payoff function $v(\cdot)$ is non decreasing in n_j.

However, as it is typical in standard bank run models, this condition does not hold, since a patient 's agent incentive to run is highest when $n_j = \frac{1}{r_1}$, rather than when it is equal to 1. Once the bank is already bankrupt, if more agents run, the probability of being paid at $t = 1$ decreases, while the $t = 2$ payment remains null; incentives to run are then lower.

In order to preserve the uniqueness result, Goldstein and Pauzner (2005) weaken the action monotonicity condition to a single crossing condition of the payoff function $v(\cdot)$. Modifying this assumption comes at the expense of a requirement on signals to be sufficiently well behaved. Noise

ε is assumed uniformly distributed (thus satisfying the Monotone Likelihood Ratio Property). As the authors point out, when departing from the uniform world, other non monotone equilibria may exist.

6.3 Proof of Proposition 1

The same arguments apply to prove that there exists one threshold strategy $x^*_{CD}(r_1)$ such that $\Delta^{r_1}(x^*_{CD}, x^*_{CD}) = 0$. First, $\Delta^{r1}(x_{ij}, x^*_{CD})$ is still continuous in $x^*_{CD}(r_1)$. Second, assume $a \in [0, \underline{\theta}_{CD}(1))$, $b \in (\overline{\theta}, 1]$, that there exists a number u such that $u \in [\Delta^{r_1}(a, x^*_{CD}), \Delta^{r_1}(b, x^*_{CD})]$ and, in particular, consider $u = 0$; (3) By the existence of lower and upper dominance range, $\Delta^{r_1}(a, x^*_{CD}) < 0$ for any $a \in [0, \underline{\theta}_{CD}(1))$ and $\Delta^{r_1}(b, x^*_{CD}) > 0$ for any $b \in (\overline{\theta}, 1]$. By the intermediate value theorem, there exists at least one threshold equilibrium $x^*_{CD} \in [0, 1]$, such that,

$$\Delta^{r_1}(x^*_{CD}, x^*_{CD}) = 0.$$

Finally, concerning uniqueness, given that single crossing condition of $v(\cdot)$ is preserved, $x^*_{CD}(r_1)$ is exactly the unique threshold equilibrium .

6.4 Proof of Proposition 2

We start by assuming that $R_f < r_1 < R$ and $p(\theta_j) = \theta_j$. The proof has three steps. First, we consider the situation with no cross deposits. From the marginal depositor's indifference condition, we obtain the equation that solves for $x^*_{NC}(r_1)$. Second, we derive the condition that solves for $x^*_{CD}(r_1)$ with cross deposits. Third, we conjecture that $x^*_{NC}(r_1) = x^*_{CD}(r_1)$. We prove by contradiction that in equilibrium it can not be true that the marginal depositor is indifferent at $x_{ij} = x^*_{NC}(r_1) = x^*_{CD}(r_1)$. We conclude that the threshold equilibrium $x^*_{CD}(r_1)$ is smaller than $x^*_{NC}(r_1)$.

6.4.1 No cross deposits

The marginal depositor's posterior distribution of θ_j is uniform over the interval $[x^*_{NC}(r_1) - \varepsilon, x^*_{NC}(r_1) + \varepsilon]$. In addition, her posterior distribution of n is uniform over $[\mu, 1]$[21]. The inverse of $n(\theta_j, x^*_{NC}(r_1))$ becomes:

$$\theta(x^*_{NC}(r_1), n) = x^*_{NC}(r_1) + \varepsilon(1 - 2\frac{n-\mu}{1-\mu}) \tag{24}$$

The equation that determines x^*_{NC} is:

$$\begin{split} f^{NC}(x^*_{NC}, r_1) \equiv \int_{n=\mu}^{1/r_1} \left(x^*_{NC} + \varepsilon(1 - 2\tfrac{n-\mu}{1-\mu}) \right) \times U(\tfrac{1-nr_1}{1-n}R)dn - \int_{n=\mu}^{1/r_1} U(r_1)dn - \\ \int_{n=1/r_1}^{1} \tfrac{1}{nr_1} U(r_1)dn = 0 \end{split} \tag{25}$$

[21] Since all agents share i) the threshold strategy x^*_{NC} and ii) uniform beliefs about the proportion of individuals who run as a function of θ_j, $n(\theta_j, x^*_{NC})$.

6.4.2 Cross deposits

Because of Lemma 8, the marginal depositor's posterior distribution of θ_j continues to be uniform, now over the interval $[x^*_{CD}(r_1)-\varepsilon, x^*_{CD}(r_1)+\varepsilon]$; her posterior distribution of n is still uniform over $[\mu, 1]$, while the inverse of $n(\theta_j, x^*_{CD}(r_1))$ is still governed by equation (24), after replacing $x^*_{NC}(r_1)$ by $x^*_{CD}(r_1)$. The equation that solves for $x^*_{CD}(r_1)$ is:

$$\begin{aligned} f^{CD}(x^*_{CD}, r_1) \quad &\equiv \quad \int_{n=\mu}^{1/r_1} \left(x^*_{CD} + \varepsilon(1 - 2\frac{n-\mu}{1-\mu}) \right) \times \left(qU(\frac{1-nr_1}{1-n}R) + (1-q)U\left(\frac{1-y}{1-n}R + \frac{y-nr_1}{1-n}R_f\right)\right) dn \\ &- \int_{n=\mu}^{1/r_1} U(r_1)dn - \int_{n=1/r_1}^{1} \frac{1}{nr_1}U(r_1)dn = 0 \end{aligned} \tag{26}$$

The marginal depositor observing $x_{ij} = x^*_{CD}(r_1)$ is indifferent between withdrawing at $t = 2$ or $t = 1$.

6.4.3 Conjecture $x^*_{NC}(r_1) = x^*_{CD}(r_1)$

If the conjecture were true, (25) would still hold, while (26) would become:

$$\begin{aligned} f^{CD}(x^*_{NC}, r_1) \quad &\equiv \quad \int_{n=\mu}^{1/r_1} \left(x^*_{NC} + \varepsilon(1 - 2\frac{n-\mu}{1-\mu}) \right) \times \left(qU(\frac{1-nr_1}{1-n}R) + (1-q)U\left(\frac{1-y}{1-n}R + \frac{y-nr_1}{1-n}R_f\right)\right) dn \\ &- \int_{n=\mu}^{1/r_1} U(r_1)dn - \int_{n=1/r_1}^{1} \frac{1}{nr_1}U(r_1)dn = 0 \end{aligned} \tag{27}$$

Under $x^*_{NC} = x^*_{CD}$, the only difference between (25) and (27) would be the first term.

We proceed to show the contradiction. If x^*_{NC} is the threshold equilibrium, contingent on bank j choosing not to cross deposit,

$$f^{NC}(x^*_{NC}, r_1) = 0 \text{ at } x_{ij} = x^*_{NC}. \tag{28}$$

Under the conjecture, it should also be true that,

$$f^{CD}(x^*_{CD}, r_1) = f^{CD}(x^*_{NC}, r_1) = 0.$$

However, for plausible values of y, namely $\underline{y} < y < \overline{y}$, [22,23]and provided $1 < R_f < r_1 < R$,

[22]In particular, provided $p(\theta_j) = \theta_j$, $\overline{y}$ solves for,

$$\int_{n=\mu}^{1/r_1} \left(\frac{1-\overline{y}}{1-n}R + \frac{\overline{y}-nr_1}{1-n}R_f\right) dn = \int_{n=\mu}^{1/r_1} \left(\frac{1-nr_1}{1-n}R\right) dn.$$

which results in $\overline{y} \equiv r_1 + \frac{1-\mu r_1}{\log(\frac{r_1-1}{r_1(1-\mu)})}$.

[23]$\overline{y}$ can also be interpreted as the level of reserves above which the expected marginal benefit from cross depositing at $t = 1$ (namely, the possibility to reallocate reserves at $t = 1$) is smaller than the marginal opportunity cost of holding those reserves at $t = 1$ (namely, the forgone return of the long term investment).

simulations show that,

$$\int_{n=\mu}^{1/r_1} \left(x^*_{NC} + \varepsilon(1 - 2\frac{n-\mu}{1-\mu}) \right) \times \left(qU(\frac{1-nr_1}{1-n}R) + (1-q)U\left(\frac{1-y}{1-n}R + \frac{y-nr_1}{1-n}R_f \right) \right) dn \tag{29}$$

$$> \int_{n=\mu}^{1/r_1} \left(x^*_{NC} + \varepsilon(1 - 2\frac{n-\mu}{1-\mu}) \right) \times U(\frac{1-nr_1}{1-n}R) dn,$$

which reflects the contradiction. It is impossible that the marginal depositor is indifferent between withdrawing at $t = 2$ or $t = 1$, at $x_{ij} = x^*_{NC} = x^*_{CD}$.

Note that $1 < R_f < r_1 < R$ is to guarantee that the marginal depositor is at least indifferent about bank's choice to participate in the interbank market at $t = 0$.

To restore the equality in (29), that is,

$$f^{CD}(x^*_{CD}, r_1) = f^{NC}(x^*_{NC}, r_1) = 0, \tag{30}$$

it has to be that,

$$x^*_{CD}(r_1) < x^*_{NC}(r_1),$$

which concludes the proof of Corollary 1.

Example. Assuming $q = 0$ and that the agents' utility function is,

$$U(c_1 + \lambda_i c_2) = \frac{(c_1 + \lambda_i c_2)^{1-\rho}}{1-\rho}; \tag{31}$$

the indifference condition for the marginal depositor in bank j, provided bank j chooses not to participate, becomes:

$$f(x^*_{NC}, r_1) = \int_{n=\mu}^{1/r_1} \left(x^*_{NC} + \varepsilon(1 - 2\frac{n-\mu}{1-\mu}) \right) \frac{1}{1-\rho} \left(\frac{1-nr_1}{1-n}R \right)^{1-\rho} dn + \tag{32}$$

$$+ \int_{n=\mu}^{n=1/r_1} \frac{1}{1-\rho} (r_1)^{1-\rho} dn + \int_{n=1/r_1}^{1} \frac{1}{nr_1} \frac{1}{1-\rho} (r_1)^{1-\rho} = 0,$$

which results in:

$$x^*_{NC}(r_1) = \frac{\frac{1}{1-\rho}(r_1)^{1-\rho}\left(\frac{1}{r_1} - \mu + \frac{\ln(r_1)}{r_1} \right) - \int_{n=\mu}^{1/r_1} \varepsilon \left(1 - 2\frac{n-\mu}{1-\mu} \right) \frac{1}{1-\rho} \left(\frac{1-nr_1}{1-n}R \right)^{1-\rho} dn}{\int_{n=\mu}^{1/r_1} \frac{1}{1-\rho} \left(\frac{1-nr_1}{1-n}R \right)^{1-\rho} dn}. \tag{33}$$

Instead, if bank j chooses to participate, the indifference condition for the marginal depositor in bank j, provided bank j chooses not to participate, becomes:

$$f(x^*_{CD}, r_1) = \int_{n=\mu}^{1/r_1} \left(x^*_{CD} + \varepsilon(1 - 2\frac{n-\mu}{1-\mu}) \right) \frac{1}{1-\rho} \left(\frac{1-y}{1-n}R + \frac{y-nr_1}{1-n}R_f \right)^{1-\rho} dn + \tag{34}$$

$$+\int_{n=\mu}^{n=1/r_1} \frac{1}{1-\rho} (r_1)^{1-\rho} \, dn + \int_{n=1/r_1}^{1} \frac{1}{nr_1} \frac{1}{1-\rho} (r_1)^{1-\rho} = 0$$

which yields,

$$x^*_{CD}(r_1) = \frac{\frac{1}{1-\rho}(r_1)^{1-\rho}\left(\frac{1}{r_1} - \mu + \frac{\ln(r_1)}{r_1}\right) - \int_{n=\mu}^{1/r_1} \varepsilon \left(1 - 2\frac{n-\mu}{1-\mu}\right) \frac{1}{1-\rho}\left(\frac{1-y}{1-n}R + \frac{y-nr_1}{1-n}R_f\right)^{1-\rho} dn}{\int_{n=\mu}^{1/r_1} \frac{1}{1-\rho}\left(\frac{1-y}{1-n}R + \frac{y-nr_1}{1-n}R_f\right)^{1-\rho} dn}. \tag{35}$$

6.5 Proof of Proposition 3

To show that $r_1^{CD} > 1$, we first compute the derivative of the expected utility of a representative depositor if bank j can cross deposit, with respect to r_1^{CD}. Next, we evaluate it at $r_1^{CD} = 1$ to show that the derivative is strictly positive for ε and $\theta^*_{CD}(1)$ sufficiently small.

With cross deposit, the expected utility of a representative depositor is:

$$\Omega^{CD} = \max_{r_1^{CD}} \left\{ \begin{array}{c} \int_0^{\theta^*_{CD}(r_1^{CD})} \frac{1}{r_1^{CD}} U(r_1^{CD}) d\theta_j \\ + \int_{\theta^*_{CD}(r_1^{CD})}^{x^*_{CD}+\varepsilon} \left(n_j U(r_1^{CD}) + (1-n_j)p(\theta_j)\left(qU\left(\frac{1-n_j r_1^{CD}}{1-n_j}R\right) + (1-q)U\left(\frac{1-y}{1-n_j}R + \frac{y-n_j r_1^{CD}}{1-n_j}R_f\right)\right)\right) d\theta_j \\ + \int_{x^*_{CD}(r_1^{CD})+\varepsilon}^{\overline{\theta}} \left(\mu U(r_1^{CD}) + (1-\mu)p(\theta_j)\left(qU\left(\frac{1-\mu r_1^{CD}}{1-\mu}R\right) + (1-q)U\left(\frac{1-y}{1-\mu}R + \frac{y-\mu r_1^{CD}}{1-\mu}R_f\right)\right)\right) d\theta_j \\ + \int_{\overline{\theta}}^{1} \left(\mu U(r_1^{CD}) + (1-\mu)\left(qU\left(\frac{R-\mu r_1^{CD}}{1-\mu}\right) + (1-q)U\left(\frac{R-y}{1-\mu} + \frac{y-\mu r_1^{CD}}{1-\mu}R_f\right)\right)\right) d\theta_j \end{array} \right\} \tag{36}$$

with n_j abbreviating for $n(\theta_j, x^*_{CD}(r_1^{CD}))$.

By Leibniz rule, the derivative of the first term in (36) is:

$$\int_0^{\theta^*_{CD}(r_1^{CD})} \frac{U'(r_1^{CD})r_1^{CD} - U(r_1^{CD})}{r_1^{CD2}} d\theta_j + \frac{\partial \theta^*_{CD}(r_1^{CD})}{\partial r_1^{CD}} \frac{U(r_1^{CD})}{r_1^{CD}},$$

which at $r_1^{CD} = 1$ becomes

$$\int_0^{x^*_{CD}(1)-\varepsilon} (U'(1) - U(1)) d\theta_j < 0,$$

since $\theta^*_{CD}(1) = x^*_{CD}(1) - \varepsilon$ and $\frac{\partial \theta^*_{CD}(1)}{\partial r_1^{CD}} = 0$, by definition of $\theta^*_{CD}(1)$.

Regarding the second term, its derivative with respect to r_1^{CD} is,

$$\begin{array}{c} \int_{\theta^*_{CD}(r_1^{CD})}^{x^*_{CD}+\varepsilon} n_j \left(U'(r_1^{CD}) - Rqp(\theta_j)U'\left(\frac{1-n_j r_1^{CD}}{1-n_j}R\right) - R_f(1-q)p(\theta_j)U'\left(\frac{1-y}{1-n_j}R + \frac{y-n_j r_1^{CD}}{1-n_j}R_f\right)\right) d\theta_j \\ + \frac{\partial x^*_{CD}(r_1^{CD})}{\partial r_1^{CD}} \left(\mu U(r_1^{CD}) + (1-\mu)p(\theta_j)\left(qU\left(\frac{1-\mu r_1^{CD}}{1-\mu}R\right) + (1-q)U\left(\frac{1-y}{1-\mu}R + \frac{y-\mu r_1^{CD}}{1-\mu}R_f\right)\right)\right) \\ - \frac{\partial \theta^*_{CD}(r_1^{CD})}{\partial r_1^{CD}} U(r_1^{CD}). \end{array} \tag{37}$$

At $r_1^{CD} = 1$, $\frac{\partial x^*_{CD}(1)}{\partial r_1^{CD}} = \frac{\partial \theta^*_{CD}(1)}{\partial r_1^{CD}} = 0$ and (37) equals

$$\int_{x^*_{CD}(1)-\varepsilon}^{x^*_{CD}(1)+\varepsilon} \mu \left(U'(1) - qRp(\theta_j)U'(R) - (1-q)R_f p(\theta_j)U'\left(\frac{1-y}{1-\mu}R + \frac{y-\mu}{1-\mu}R_f\right)\right) d\theta_j. \tag{38}$$

If $q = 1$, it is easy to show that (38) is

$$\int_{x^*_{CD}(1)-\varepsilon}^{x^*_{CD}(1)+\varepsilon} \left(U'(1) - Rp(\theta_j)U'(R)\right) > 0,$$

since $U'(1) > RU'(R)$ for all $R > 1$ and because $p(\theta_j) < 1$. Hence, it suffices to show that (38) is still strictly positive if $q = 0$.

If $q = 0$, provided the utility function is sufficiently well behaved and assuming that $R > 1$, $R_f > 1$ and y are such that $\int_{x^*_{CD}(1)-\varepsilon}^{x^*_{CD}(1)+\varepsilon} p(\theta_j)\left(\frac{1-y}{1-\mu}R + \frac{y-\mu}{1-\mu}R_f\right) d\theta_j > 1$, (38) satisfies,

$$\int_{x^*_{CD}(1)-\varepsilon}^{x^*_{CD}(1)+\varepsilon} U'(1) - R_f\left(p(\theta_j)U'\left(\frac{1-y}{1-\mu}R + \frac{y-\mu}{1-\mu}R_f\right)\right) d\theta_j > 0. \tag{39}$$

We conclude that (38) is positive for all possible value of q.

Reasoning similarly, we can show that the derivative of the third and fourth term in (36), with respect to r_1^{CD} are also strictly positive at $r_1^{CD} = 1$.

Recapitulating, the derivative of (36) with respect to r_1^{CD} at $r_1^{CD} = 1$ equals,

$$\begin{gathered} \int_0^{\theta^*_{CD}(1)}(U'(1) - U(1))d\theta_j + \int_{\theta^*_{CD}(1)}^{x^*_{CD}(1)+\varepsilon} \mu \\ \left(U'(1) - Rqp(\theta_j)U'(R) - R_f(1-q)p(\theta_j)U'\left(\tfrac{1-y}{1-\mu}R + \tfrac{y-\mu}{1-\mu}R_f\right)\right) d\theta_j \\ + \int_{x^*_{CD}(1)+\varepsilon}^{\overline{\theta}} \mu\left(U'(1) - qRp(\theta_j)U'(R) - (1-q)R_f p(\theta_j)U'\left(\tfrac{1-y}{1-\mu}R + \tfrac{y-\mu}{1-\mu}R_f\right)\right) d\theta_j \\ + \int_{\overline{\theta}}^1 \left(\mu U'(1) + (1-\mu)\left(qRU'\left(\tfrac{R-\mu}{1-\mu}\right) + (1-q)R_f U'\left(\tfrac{R-y}{1-\mu} + \tfrac{y-\mu}{1-\mu}R_f\right)\right)\right) d\theta_j \end{gathered} \tag{40}$$

Terms 2 to 4 in (40) are positive, while the first term is negative. This term, however, is small for sufficiently small $\theta^*_{CD}(1) = x^*_{CD}(1) - \varepsilon$. As ε goes to 0, $\theta^*_{CD}(1)$ and $x^*_{CD}(1)$ converge to $\underline{\theta}_{CD}(1)$. Thus, for a sufficiently small $\underline{\theta}_{CD}(1)$, the claim of the proposition holds.

6.6 Derivation of Inequality (17)

First, provided $y > \mu$ and $q < 1$, the lower bound of fundamentals without and with interbank market, $\underline{\theta}_{NC}(r_1)$ and $\underline{\theta}_{CD}(r_1)$, respectively, satisfy

$$\underline{\theta}_{CD}(r_1) > \underline{\theta}_{NC}(r_1). \tag{41}$$

The level of fundamentals below which a patient agent prefers to withdraw, no matter what other agents do, increases with an interbank market. This is because the expected return if waiting until $t = 2$ is lower with cross deposits, under these conditions. Instead, if $q = 1$, $\underline{\theta}_{CD}(r_1) = \underline{\theta}_{NC}(r_1)$.

Second, we have shown that

$$\theta^*_{CD}(r_1) < \theta^*_{NC}(r_1)$$

and that for any $r_1 > 1$, $\theta^*_{CD}(r_1)$, $\theta^*_{NC}(r_1)$, $\underline{\theta}_{CD}(r_1)$ and $\underline{\theta}_{NC}(r_1)$ increase with r_1.

Hence, equation (17) holds.

6.7 Proof of Proposition 4

Lemma 5 and proposition 3 show that $1 < r_1^{NC} < c_1^{FB}$ and $1 < r_1^{CD} < c_1^{FB}$. Hence, what we need to prove is that $r_1^{CD} \geq r_1^{NC}$.

To do this, we start by recalling proposition 2 and equation (16), which state that iff $r_1^{NC} = r_1^{CD} = r_1$ and provided $\underline{y} < y < \overline{y}$ and $1 < R_f < r_1 < R$,

$$0 < \theta^*_{CD}(r_1) < \theta^*_{NC}(r_1) < 1.$$

Because banks only participate in the interbank market iff $\theta^*_{CD}(r_1^{CD}) \leq \theta^*_{NC}(r_1^{NC})$ and since both $\theta^*_{CD}(r_1^{CD})$ and $\theta^*_{NC}(r_1^{NC})$ are increasing in r_1^{CD} and r_1^{NC}, respectively, in equilibrium, it is true that,

$$r_1^{CD} \geq r_1^{NC}.$$

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