Credit and Liquidity in Interbank Rates:
a Quadratic Approach

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Abstract

In this paper, we propose a quadratic term-structure model of the EURIBOR-OIS spreads. These spreads are affected by both credit and liquidity risks, which we aim at disentangling. Liquidity effects reflect banks preferring a portfolio of cash and easy-to-liquidate swap contracts to interbank loans, to cope with potential future liquidity needs. Credit effects correspond to the premium required by the lender for the borrower's default risk compensation. Our approach allows us to identify credit and liquidity effects and to further decompose the whole term structure of spreads into credit- and liquidity-related parts. Our results shed new light on the effects of unconventional monetary policy carried out in the Eurosystem. In particular, our findings suggest that most of the recent easing in the euro interbank market is liquidity related.

JEL Codes: E43, E44, G12, G21

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1 Introduction

Since the beginning of the financial crisis, the interbank market has been carefully scrutinized by commentators and policy-makers, both in Europe and in the US. This paper focuses on the spread between the rates on unsecured interbank loans (e.g. EURIBOR or LIBOR) and their risk-free counterparts, proxied by the Overnight Indexed Swap rate (OIS). This spread is considered as a crucial indicator at the very core of the financial crisis: it reveals banks' concerns regarding both the credit risk of their counterparts and their own liquidity needs.

In this paper, we disentangle credit and liquidity effects in the European interbank market. This decomposition possesses essential policy implications. The appropriate actions to address a sharp rise in spreads strongly depend on its cause: if the rise in spreads reflects poor liquidity, policy measures should aim at improving funding facilities. On the other hand credit concerns should be treated by enhancing debtors' solvency (see Codogno et al. [2003]). This question is of utmost importance in the euro area, where most of the unconventional monetary operations conducted by the European Central Bank aim at the curbing of interbank risk (see Gonzales-Paramo [2011]).[1]

Our paper comes within the scope of the literature on term structure models of interest rates. We build a two-factor arbitrage-free quadratic term structure model (QTSM) to reproduce the dynamics of the term structure of EURIBOR-OIS spreads. The quadratic specification of our model features several useful properties: high tractability with closed-form spread formulae and strict positivity of modelled spreads – a challenging task for classic ATSM.[2] In addition, this representation of yields as quadratic functions of factors makes it possible to capture higher-order moments of interbank spreads.

In our framework, the spreads are functions of credit and liquidity factors. The identification of the two shocks relies on credit and liquidity proxies. This allows us to separate spread fluctuations attributable to liquidity and credit shocks. In addition, no-arbitrage assumptions allow for the EURIBOR-OIS term structure decomposition into an expectation part and a risk premium part. The former should be interpreted as the spread that would have prevailed if market participants were risk-neutral, whereas the latter depends on the aversion of market participants to default and liquidity risks.

Hence we obtain a double decomposition of interbank spreads – credit/liquidity on the one hand and expectations/risk premium on the other hand – which helps to identify the consequences of unconventional monetary policies conducted by the ECB. We find that the credit component features low-frequency fluctuations, and shows a persistent increase from August 2007 before stabilizing in August 2012. The liquidity component experiences higher-frequency variations; in particular, it has monotonously dropped since late December 2011. Eventually, the liquidity part of the spreads is economically negligible in January 2013. Thus, our results suggest that the recent 3-year ECB loans to euro commercial banks and the recently-announced ECB bond purchase program have helped to

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[1] The ECB has changed dramatically its operational framework to counterbalance the interbank markets freeze, by conducting special refinancing operations with longer-than-usual maturity, or by establishing a *fixed rate full allotment* rule to provide unlimited amount of liquidity to euro commercial banks at fixed cost.

[2] See Kim and Singleton [2012] for an example of QTSM for the pricing of Japanese government bonds, when the interest rates are close to the zero lower bound.

reduce the perception of liquidity risk and its related premium.[3]

The remainder of the paper is organized as follows. Section 2 presents the related literature. Section 3 details the construction of interbank rates. Section 4 develops the quadratic term-structure model. Section 5 describes the identification strategy and shows the estimation results. Section 6 performs the decomposition of EURIBOR-OIS spreads and discusses the impact of the ECB unconventional monetary policies. The last section concludes. Proofs are gathered in the Appendices.

2   Literature Review

In most term structure models, the authors assume that the intensity or the short-term rate is an affine combination of the underlying factors. A quadratic specification however possesses several advantages. Constantinides [1992] shows that a standard term structure with a specific quadratic short-term interest rate can generate positive yields for all maturities, and more flexibility in the term structure to fit bond data. Leippold and Wu [2002a] generalize the quadratic short-rate term structure models showing that this specification provides closed-form or semi closed-form formulae for bond pricing of most fixed-income derivatives. Leippold and Wu [2002b] provide further empirical evidence that QTSM often outperforms the standard affine term structure specification (ATSM).

Our identification scheme follows several recent studies that model yield curves associated with different fixed-income instruments (e.g. bonds, repo, swaps). These studies usually exploit this modeling to breakdown credit spreads or swap spreads into different components. Specifically, Liu, Longstaff, and Mandell [2006] use a five-factor affine framework to jointly model Treasury, repo and swap term structures. One factor is related to the pricing of the Treasury-securities liquidity and another one reflects default risk. Feldhutter and Lando [2008] develop a six-factor model for Treasury bonds, corporate bonds and swap rates. They decompose swap spreads into three components: a convenience yield from holding Treasuries, a credit-element associated with the underlying LIBOR rate, and a factor specific to the swap market. Their results indicate that the convenience yield interpreted as liquidity premium is by far the largest component of spreads. Longstaff, Mithal, and Neis [2005] use information in credit default swaps in addition to bond prices to obtain measures of the non-default components in corporate spreads. Their estimation suggests that the non-default component is time-varying and strongly related to measures of bond-specific illiquidity as well as to macroeconomic measures of bond-market liquidity. Monfort and Renne [2012b] show that a substantial part of euro-area sovereign spreads are driven by a liquidity component. The identification of the latter relies on the interpretation of the spreads between the bonds issued by KfW, a public German agency, and their sovereign counterparts. Indeed, since KfW bonds are fully and explicitly guaranteed by the Federal Republic of Germany, these spreads should essentially reflect liquidity-pricing effects.

This paper is also related to the literature that focuses on interbank spreads. A wide range of studies deals with the determinants of interbank spreads: Taylor and Williams [2009] claim that counterparty risk was the main driver of the LIBOR/OIS spread, Michaud and Upper [2008] and Gyntelberg and

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[3]We refer to the 3-year ECB loans to euro commercial banks as *Very Long-Term Refinancing Operations* (VLTRO) and to the ECB bond purchase program as *Outright Monetary Transactions* (OMT).

Wooldridge [2008] find that credit and liquidity factors both played a role, while the results by Schwarz [2009] and Filipovic and Trolle [2011] suggest that liquidity risk has accounted for most of the LIBOR/OIS and EURIBOR/OIS spread variations over the period 2007-2009. Finally, Angelini, Nobili, and Picillo [2011] highlight the main role of macro-factors – such as the aggregate risk-aversion as opposed to individual lenders' and borrowers' characteristics – to account for the dynamics of unsecured/secured money-market spreads. The measured impact of unconventional monetary policies is also ambiguous: Taylor and Williams [2009] find no effects of the Fed's intervention in 2008, contrary to Christensen, Lopez, and Rudebusch [2009]. According to the latter, the Fed's TAF reduced significantly the 3-month maturity interbank spread by about 70 basis points.[4] In Europe, Angelini, Nobili, and Picillo [2011] measure a modest impact of ECB exceptional 3-month refinancing operations, in contradiction with Abbassi and Linzert [2011].[5]

3 Interbank market rates and risks

3.1 The unsecured interbank rates

The interbank money market is at the heart of bank funding issues. It is an over-the-counter market (OTC) where interbank loans are negotiated with maturities ranging from one day to to 12 months. As banks do not possess the same characteristics and underlying risks, there is no uniqueness of interbank rates. Only the disaggregated rates are really representative of the funding issues of each institution. However, such data are not publicly available.[6] In order to conduct an analysis on interbank risks, a more aggregated measure must be considered.

The Euro Interbank Offered Rate (EURIBOR) provides a measure of the interest rate at which banks can raise unsecured funds from other financial institutions.[7] The European Banking Federation publishes a daily reference rate based on the trimmed averaged interest rates at which Eurozone banks offer to lend unsecured funds to other banks in the euro wholesale money market. There is one rate for each maturity between one week and twelve months. More specifically, a daily survey is sent to a panel of 40 to 50 banks in the Euro area. The selected banks are those with supposedly high creditworthiness. The question of the survey is *what are the rates at which euro interbank term deposits are being offered within the Eurozone by one prime bank to another.* Contrary to the LIBOR survey (US), the banks are not asked about their own situation. The trimmed mean erases the 15% banks

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[4] The TAF (for *Term Auction Facility*) enabled the US Federal Reserve to refund depository institutions against a wide range of collateral. The program aimed to provide a wider access to the Fed liquidity to financial institutions, in a period of heightened concerns regarding the liquidity needs of financial institutions.

[5] See Cecioni, Ferrero, and Secchi [2011] for a review of (a) the quantitative assessment regarding the relative importance of the interbank spread drivers, and (b) of the effects of unconventional monetary policies in the USD and euro interbank market.

[6] Notwithstanding, the individual contributions of panel banks are available at http://www.euribor-ebf.eu/euribor-org However, given the specific question that is posed to the banks (see below), their contributions do not necessarily reveal their own lending or borrowing costs.

[7] Temporary cases of frauds on both LIBOR and EURIBOR declarations by individual banks have been revealed by traders' communications disclosure. In particular, Barclay's Bank was charged $200 million by the Commodity Futures Trading Commission, $160 million by the American Department of Justice, and $59.5 million by the Financial Services Authority. However, there is no information on both (i) how those figures have been determined, (ii) the overall bias of LIBOR declarations. Using CDS data, Mollenkamp and Whitehouse [2008] evaluate the difference between the reported rate and the actual rate ranging from 3 to 87 bps depending on the institution. Nonetheless, contrary to LIBOR, Eisl, Jankowitsch, and Subrahmanyam [2013] show that EURIBOR rates are less likely to be manipulated and less exposed in size to those frauds. Such considerations on the precise evaluation of misleading declarations are beyond the scope of this paper.

of each distribution tail.

The loans that underlie the EURIBOR are unsecured, that is,the lending bank does not receive collateral as protection against default by the borrowing one. Therefore, these rates carry some compensation for solvency issue that we refer to as credit risk. Furthermore, through an interbank loan, a lending bank exposes its funds during the time-to-maturity of the loan although those funds might be needed to cover the bank's own shortfalls (see e.g. Taylor and Williams [2009] or Michaud and Upper [2008]). Moreover, since an unsecured interbank loan is highly specific to the identity of both counterparties, its unwinding is a costly task. Thus the liquidity risk affects the rate at which this bank is willing to lend.[8]

Figure 1 presents the evolution of the 3-month EURIBOR from August 2007 to January 2013. During the first year, the rate is stable around 500 basis points. The Lehman bankruptcy of September 2008 is followed by a sharp decline in EURIBOR of about of about 400 basis points, to 80 basis points. From mid-2010 onwards, the EURIBOR rises slowly to 150 basis points in September 2011 and decays to nearly 20 basis points during the recent period. Table 1 presents the descriptive statistics for 3, 6, 9, and 12-month EURIBOR maturities.

Figure 1: Level of 3M rates and spreads
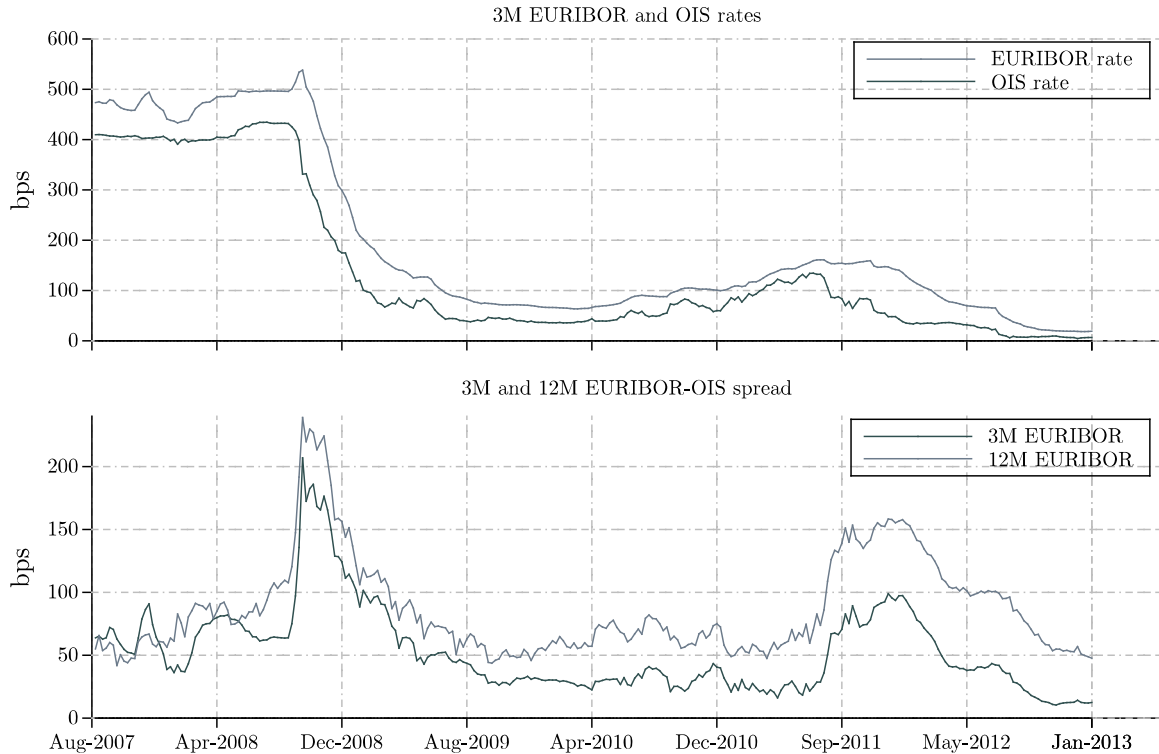
Notes: Top panel: plot of the 3M EURIBOR (dark grey) and 3M OIS (lighter grey). Bottom panel: plot of the 3M (dark grey) and 12M (lighter grey) EURIBOR-OIS spreads. Units are in basis points.

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[8]This liquidity risk encompasses both market and funding liquidity issues. These are known to be difficult to assess separately. Brunnermeier and Pedersen [2009] define *market liquidity* as the difference between market and fundamental values of an asset, and *funding liquidity* as "speculator's scarcity of capital". They show, in particular, that the covariance between market and funding liquidity is positive, and that illiquidity spirals can arise, causing market illiquidity to impact crucially speculators' funding illiquidity through higher margins.

3.2 The interbank *risk-free* rate

In this paper, the risk-free rates are proxied by the Overnight Indexed Swap (OIS) rates. An OIS is a fixed-for-floating interest rate swap with a floating rate leg indexed on overnight interbank rates, the EONIA in the euro-area case. OIS have become especially popular hedging and positioning vehicles in euro financial markets and grew significantly in importance during the financial turmoil of the last few years. The OIS curve is more and more seen by market participants as a proxy of the risk-free interbank yield curve (see e.g. Joyce, Lasaosa, Stevens, and Tong [2011]). As no principal is exchanged, the OIS requires nearly no immobilisation of capital. Further, due to netting and credit enhancement mechanisms (including call margins), the counterparty risk is limited in the case of a swap contract (see Bomfim [2003]): it reduces the risk of loss due to the default of the borrower and facilitates the search for reverse contracts to close the lender's position before the swap expiry date.

The upper panel of Figure 1 displays the 3-month OIS rate from August 2007 to January 2013. While this chart shows that EURIBOR and OIS rates present strong common fluctuations, it also highlights that the spread between the two rates has undergone substantial variations over the last five years. In the next subsection, we discuss the term structure of the EURIBOR-OIS spreads.

3.3 Preliminary analysis of the EURIBOR-OIS spreads

Being mostly stable before August 2008, the spread increased abruptly during Lehman crisis until December 2008, the 3-month spread peaking at 200 basis points, where a slow decay begins (see Figure 1, bottom).[9] Then, following a long stabilization period between August 2009 and 2010, a sharp rise stroke again in mid-2011. Since the beginning of 2012, the EURIBOR-OIS spreads have decreased, alternating between a linear decreasing trend and stable plateaux.

Table 1: Descriptive statistics of EURIBOR and OIS rates

	min	max	amplitude *bps*	mean	std	skewness	excess kurtosis
EURIBOR 3M	18,4	538,1	519,7	191,3	165,7	1,00	−0,69
EURIBOR 6M	31,6	543,1	511,5	211,1	157,7	1,00	−0,63
EURIBOR 9M	42,6	546,3	503,7	222,7	153,3	1,00	−0,60
EURIBOR 12M	53,7	549,3	495,6	233,5	149,8	0,99	−0,56
OIS 3M	4,5	434,6	430,1	138,3	148,5	1,14	−0,45
OIS 6M	2,35	442,85	440,5	140,0	147,6	1,15	−0,39
OIS 9M	−0,5	453,5	454	142,9	146,0	1,14	−0,35
OIS 12M	−1,1	465,3	466,4	146,5	144,1	1,13	−0,32
Spread 3M	10,4	206,9	196,5	53,0	34,4	1,64	3,44
Spread 6M	26,15	222,5	196,35	71,1	35,3	1,78	3,83
Spread 9M	37,45	227,9	190,45	79,8	37,3	1,72	3,17
Spread 12M	41,82	239	197,18	87,0	39,5	1,55	2,29

Notes: Those figures are computed with weekly data ranging from 31st August 2007 to 4th January 2013.

Standard descriptive statistics of spreads are provided in Table 1. The OIS average for different maturities is between 50 and 90 basis points below the EURIBOR averages. It is also less volatile and the volatility is similar across maturities of OIS. In comparison, the volatility of EURIBOR rates

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[9]For sake of comparison, before summer 2007, the EURIBOR-OIS spread was around ten basis points. Part of this deviation was accounted for by the fact that the EURIBOR is an offer rate while the OIS is a mid rate.

decreases more deeply with maturity. The means of spreads increase with respect to maturity, from 53 to 87 basis points. This indicates a positive slope in the term structure of spreads, that is graphically illustrated by the bottom panel in Figure 1: except at the very beginning of the sample, the 12-month spread is always larger than the 3-month spread, up to around 50 basis points in late 2011. Furthermore, the same plot shows that the slope is time-varying.

Whereas the standard deviations are decreasing with maturity for EURIBOR and OIS rates, the standard deviations of spreads increase with maturity. Regarding higher-order moments, Table 1 indicates that both EURIBOR and OIS rates for all maturities are positively skewed and possess thin tail distributions (negative excess kurtosis). For all maturities, spreads are more positively skewed than the rates in level; also, contrary to the latter, spreads are heavy-tailed (positive excess kurtosis). The heavy-tail behavior is typically illustrated during the Lehman crisis on Figure 1, where both 3-month and 12-month spreads peak to 207 and 239 basis points, respectively. These levels are about 4-standard-deviation far from their respective sample means.

Finally, a principal component analysis performed on the four EURIBOR-OIS spreads proves that the first principal component captures most of spread fluctuations. It explains nearly 96% of the whole variance of the spreads, emphasizing the very similar patterns observed in the variations of the spreads of different maturities.

In the next section, we develop a model that is consistent with these observations.

4 The model

4.1 The intensity

At date t, market participants get the new information $w_t = \{r_t, X_t, d_t\}$, where r_t is the short-term risk-free rate between dates t and $t+1$, $X_t = [x_{c,t}, x_{l,t}]'$ is a 2×1 vector whose components are respectively a credit-risk factor and a liquidity-risk one, and d_t is a binary variable valued in $\{0, 1\}$. A switch from $\{d_{t-1} = 0\}$ to $\{d_t = 1\}$ corresponds to one of two adverse situations from the lender point of view: either (a) the borrower on the unsecured interbank market defaults at date t, or (b) the lender on the unsecured market would have need the (lent) amount of liquidity for other purposes, which translates into costs for her.[10]

The state $\{d_t = 1\}$ is assumed to be absorbing. Let us denote by $\underline{w_t}$ the cumulated information up to date t, that is $\underline{w_t} = \{w_t, w_{t-1}, ...\}$. Conditionally on $(r_t, X_t, \underline{w_{t-1}})$, the probability of switching from $\{d_{t-1} = 0\}$ to $\{d_t = 1\}$ is given by:

$$\mathbb{P}(d_t = 1 | d_{t-1} = 0, r_t, X_t, \underline{w_{t-1}}) = 1 - \exp(-\lambda_t).$$

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[10] While the explicit modeling of such costs is beyond the scope of this paper, let us give some brief rationale behind these. In order to meet an expected liquidity need, the bank that had lent on the unsecured interbank market has two options: (a) to get funding on the interbank market or (b) to sell assets. Assuming that the state of nature $\{d_t = 1\}$ is marked by an aggregate shortfall of liquidity: the funding rate in option (a) is likely to be prohibitive; many banks are led to sell assets simultaneously, driving down the selling prices of assets, which makes option (b) costly too.

where λ_t is a function of X_t that we call intensity. Also, we assume that there is no Granger causality from d_t to (r_t, X_t).

4.2 General pricing formulae

We assume that there exists a stochastic discount factor (SDF) between t and $t+1$, which is denoted by $M_{t,t+1}$. This existence implies that the variables gathered in w_t have both physical ($\mathbb{P}$) and risk-neutral ($\mathbb{Q}$) dynamics. In addition, we assume that $M_{t,t+1}$ does not depend on d_{t+1}.

Let us denote by $R_{t,h}^{OIS}$ and $R_{t,h}^{EUR}$ the OIS rate of maturity h and the EURIBOR of the same maturity, respectively.[11] Both rates are homogeneous to zero-coupon rates. Whereas this is obvious in the EURIBOR case, let us explain why it is also the case for OIS rates. Consider a bank that has access to the overnight interbank market as well as to the EONIA swap market. Assume further that this bank wants to invest 100 euros for a period of h weeks. Then, that bank can replicate a zero-coupon investment yielding a compounded interest rate of $R_{t,h}^{OIS}$ by (a) entering a maturity-h OIS swap (where it pays the floating leg) while (b) lending, every day, the 100 euros (plus daily accrued interests) on the overnight interbank market. By definition of the OIS contract, the accrued interests earned by investing in the overnight market are going to equalize the interest accrued on the floating leg of the swap. Therefore, this strategy boils down to lending 100 euros at date t and to get $100 \exp(h/52 \times R_{t,h}^{OIS})$ at date $t+h$, which demonstrates that $R_{t,h}^{OIS}$ can be interpreted as a zero-coupon yield.

Recalling that we consider the OIS rates as risk-free yields, we have $r_t = R_{t,1}^{OIS}$ and, for longer maturities:

$$R_{t,h}^{OIS} = -\frac{1}{h} \log \left(\mathbb{E}_t^{\mathbb{Q}} \left[\exp \left\{ -r_t - \ldots - r_{t+h-1} \right\} \right] \right)$$

where $\mathbb{E}^{\mathbb{Q}}$ denotes the expectation under the risk-neutral measure, conditional on $\underline{w_t}$. Turning to the EURIBOR rates, we have:[12]

$$R_{t,h}^{EUR} = -\frac{1}{h} \log \left(\mathbb{E}_t^{\mathbb{Q}} \left[\exp \left\{ -r_t - \lambda_{t+1} - \ldots - r_{t+h-1} - \lambda_{t+h} \right\} \right] \right).$$

As in, e.g., Pan and Singleton [2008] or Longstaff, Pan, Pedersen, and Singleton [2011], we assume that the short-term risk-free interest rate is independent from the intensity λ_t. Denoting by $S(t, h)$ the EURIBOR-OIS spread of maturity h, it follows that:

$$S(t, h) = R_{t,h}^{EUR} - R_{t,h}^{OIS} = -\frac{1}{h} \log \left(\mathbb{E}_t^{\mathbb{Q}} \left[\exp \left\{ -\lambda_{t+1} - \ldots - \lambda_{t+h} \right\} \right] \right). \tag{1}$$

Equation (1) shows that, under these assumptions, the study of EURIBOR-OIS spreads does not require the modelling of short-term risk-free interest rate r_t.

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[11]The pricing formulas derived in this paper implicitly feature continuously-compounded interest rates. Let r denote a market-quoted interest rate (the OIS, say). Using the fact that the money-market day-count convention is ACT/360, the corresponding continuously-compounded rate is given by $\ln(1 + d \times r/360) \times 365/d$ where d is the residual maturity of the instrument.

[12]Note that this formula holds because (a) the SDF is assumed not to depend on d_t and (b) the latter do not Granger-cause λ_t. Moreover, these assumptions imply that the historical and risk-neutral intensities are the same processes, i.e. $\mathbb{Q}(d_t = 1 | d_{t-1} = 0, r_t, X_t, \underline{w_{t-1}}) = \mathbb{P}(d_t = 1 | d_{t-1} = 0, r_t, X_t, \underline{w_{t-1}}) = 1 - \exp(-\lambda_t)$ (see Monfort and Renne [2012a] or Gourieroux, Monfort, and Renne [mimeo]).

4.3 Intensity specification

The preliminary analysis in section 3.3 provides evidence that one factor is sufficient to account for most of the EURIBOR-OIS term structure. Hence, we assume that the intensity depends on a single common factor denoted by x_t, which is the sum of the the credit-related factor $x_{c,t}$ and the liquidity-related one $x_{l,t}$.

$$x_t = x_{c,t} + x_{l,t} \tag{2}$$

Moreover, the intensity is a quadratic function of x_t:

$$\lambda_t = \lambda_0 + \lambda_1 x_t + \lambda_2 x_t^2. \tag{3}$$

To ensure that the underlying probability is constrained between 0 and 1, λ_t has to be positive whatever the value of x_t. This constraint writes $\lambda_0 \geqslant \lambda_1^2 / 4\lambda_2$.
Under the risk-neutral measure, factor x_t follows a stationary AR(1):

$$x_t = \mu^* + \varphi^* x_{t-1} + \varepsilon_t^* \quad \text{where} \quad \varepsilon_t^* \sim \mathcal{IIN}^{\mathbb{Q}}(0,1), \quad |\varphi^*| < 1, \tag{4}$$

and the standard deviation of x_t's innovations are set to one for sake of identification.

4.4 Dynamics of $x_{c,t}$ and $x_{l,t}$

Let us now present the physical dynamics of the credit and the liquidity factors $x_{c,t}$ and $x_{l,t}$. As the two risks can influence each other (see e.g. Ericsson and Renault [2006]), we authorize lagged causality between the two factors. However, these factors are contemporaneously influenced by independent idiosyncratic shocks $\varepsilon_{c,t}$ and $\varepsilon_{l,t}$, that we refer to as credit shock and liquidity shock, respectively. Their joint dynamics is described by the following $VAR(1)$ representation.

$$\begin{pmatrix} x_{c,t} \\ x_{l,t} \end{pmatrix} = \begin{pmatrix} \mu_c \\ \mu_l \end{pmatrix} + \begin{pmatrix} \varphi_{1,1} & \varphi_{1,2} \\ \varphi_{2,1} & \varphi_{2,2} \end{pmatrix} \begin{pmatrix} x_{c,t-1} \\ x_{l,t-1} \end{pmatrix} + \begin{pmatrix} \sigma_c & 0 \\ 0 & \sigma_l \end{pmatrix} \begin{pmatrix} \varepsilon_{c,t} \\ \varepsilon_{l,t} \end{pmatrix} \tag{5}$$

where $(\varepsilon_{c,t}, \varepsilon_{l,t})' \sim \mathcal{IIN}^{\mathbb{P}}(0, I_2)$, and the eigenvalues of the autoregressive matrix $\Phi = [\varphi_{i,j}]_{i,j=\{1,2\}}$ are lesser than one in modulus.
Assuming that the SDF is exponential-affine in $(x_{c,t}, x_{l,t})'$, the expanded risk-neutral dynamics of these factors is of the form:[13]

$$\begin{pmatrix} x_{c,t} \\ x_{l,t} \end{pmatrix} = \begin{pmatrix} \mu_c^* \\ \mu_l^* \end{pmatrix} + \begin{pmatrix} \varphi_{1,1}^* & \varphi_{1,2}^* \\ \varphi_{2,1}^* & \varphi_{2,2}^* \end{pmatrix} \begin{pmatrix} x_{c,t-1} \\ x_{l,t-1} \end{pmatrix} + \begin{pmatrix} \sigma_c & 0 \\ 0 & \sigma_l \end{pmatrix} \begin{pmatrix} \varepsilon_{c,t}^* \\ \varepsilon_{l,t}^* \end{pmatrix}$$

where $(\varepsilon_{c,t}^*, \varepsilon_{l,t}^*)' \sim \mathcal{IIN}^{\mathbb{Q}}(0, I_2)$. As for the physical dynamics, the process is assumed stationary under the risk-neutral measure.
Given that we want the risk-neutral dynamics of $x_t = x_{c,t} + x_{l,t}$ to be as described by Equation (4), the parameter specifying the risk-neutral dynamics of $(x_{c,t}, x_{l,t})'$ have to satisfy:

$$\mu^* = \mu_c^* + \mu_l^* \text{ and } \varphi_{1,1}^* + \varphi_{2,1}^* = \varphi_{1,2}^* + \varphi_{2,2}^* = \varphi^* \text{ and } \sigma_c^2 + \sigma_l^2 = 1. \tag{6}$$

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[13]The SDF specification is provided in appendix A.1.

4.5 Recursive pricing formulae

Let us denote $X_t = (x_{c,t}\ x_{l,t})'$ for simplicity. It is a well-known result that Equations (3) and (4) implies that the spreads $S(t,h)$, defined by Equation 1, can be expressed as a quadratic function of X_t (compare with e.g. Leippold and Wu [2002a] in the context of quadratic short-term rate). From Equations (1), (3) and (4), we have:

$$S\left(t,h\right) \quad = \quad \theta_{0,h} + \theta_{1,h}x_t + \theta_{2,h}x_t^2 \tag{7}$$

The three parameters $\theta_{0,h}$, $\theta_{1,h}$ and $\theta_{2,h}$ are maturity-dependent and are functions of μ^*, φ^*, λ_0, λ_1, and λ_2. We further show in Appendix A.3 that the $\theta_{0,h}$, $\theta_{1,h}$, and $\theta_{2,h}$ loadings can be computed recursively as:

$$\begin{cases} -h\theta_{0,h} &= -\lambda_0 + \theta_{0,h-1} + \dfrac{\widetilde{\theta}_{1,h-1}}{K_h}\left(\mu^* + \tfrac{1}{2}\widetilde{\theta}_{1,h-1}\right) + (\mu^*)^2\dfrac{\widetilde{\theta}_{2,h-1}}{K_h} - \tfrac{1}{2}\log\left(K_h\right) \\ -h\theta_{1,h} &= \dfrac{\varphi^*}{K_h}\left(2\mu^*\widetilde{\theta}_{2,h-1} + \widetilde{\theta}_{1,h-1}\right) \\ -h\theta_{2,h} &= (\varphi^*)^2\dfrac{\widetilde{\Theta}_{h-1}}{K_h} \end{cases} \tag{8}$$

where $\widetilde{\theta}_{h-1} = \theta_{h-1} - \lambda_1$, $\widetilde{\theta}_{2,h-1} = \theta_{2,h-1} - \lambda_2$, and $K_h = (1 - 2\widetilde{\theta}_{2,h-1})$.

5 Estimation procedure

5.1 The structural identification of credit and liquidity factors

Observations of spreads are not sufficient to separate the credit factor and the liquidity one since, as shown by Equation (7), spreads depend on the sum of the factors (i.e. x_t) only. We therefore introduce credit and liquidity proxies in order to identify $x_{c,t}$ and $x_{l,t}$. Let us detail the computation of these proxies.

The liquidity proxy is the first principal component[14] of a set of three liquidity-related variables. These variables are chosen in order to capture different aspects of liquidity pricing. In particular, the first two proxies are mostly related to market liquidity whereas the last one is mostly related to funding liquidity.

- A first liquidity-pricing factor is the *KfW-Bund spread.*[15] KfW is a public German agency. KfW bonds are guaranteed by the Federal Republic of Germany. Hence, they embed the same credit quality as their sovereign counterparts, the so-called Bunds. KfW bonds being less liquid than their sovereign counterpart), the KfW-Bund spread essentially reflect liquidity-pricing effects, see Schwarz [2009], Monfort and Renne [2012a] or Schuster and Uhrig-Homburg [2012]. In the same spirit, Longstaff [2004] computes liquidity premia based on the spread between U.S. Treasuries and bonds issued by Refcorp, that are guaranteed by the Treasury.

- A second liquidity factor is the Tbill-repo spread, computed as the yield differential between the 3-month German T-bill and the 3-month general-collateral repurchase agreement rate (repo).

---

[14] nearly 50% of the total variance is explained by the first principal component.
[15] The KfW bond yield not being available for all maturities, we use the 5-year KfW-Bund spread.

From an investor point of view, the credit qualities of the two instruments are comparable (as argued by Liu, Longstaff, and Mandell [2006]). The differential between the two rates corresponds to the convenience yield, that can be seen as a premium that one is willing to pay when holding highly-liquid Treasury securities.[16]

- A third factor is based on the Bank Lending Survey conducted by the ECB on a quarterly basis[17]. Specifically, this indicator is based on the following question: *Over the past three months, how has your bank's liquidity position affected the credit standards as applied to the approval of loans or credit lines to enterprises?*[18] A weekly series is obtained by linearly interpolating the quarterly series.

The credit proxy is the first principal component of a set of 36 Euro-zone bank CDS denominated in USD.[19] Eight are German, six Italian, five Spanish, four French, four Dutch, three Irish, three Portuguese, two Austrian, and one Belgian. As a robustness check exercise, we also replace the first principal component by the median or the 50% trimmed mean of the CDS as in Filipovic and Trolle [2011]. Our results are robust to this change in credit proxy.

The dynamics of the proxies are provided on Figure 2. The liquidity proxy experiences a great peak right before the Lehman crisis, whereas the credit proxy tends to increase until late November 2011. For both proxies, we observe a particularly calm period from August 2009 to April 2010, when the interbank market became less tensed. Looking at the monetary policy events, we see that VLTRO events of December 2011 and March 2012 (the announcement and the two allotments, third to fifth vertical black line on Figure 2) are associated with a decrease of the proxies. The same result is available for Mario Draghi's London speech of late July 2012 (last vertical black line).[20]

5.2 Identification strategy: linking proxies and latent factors

We denote the credit and liquidity proxies by $P_{c,t}$ and $P_{l,t}$ respectively. We posit that the proxies are quadratic functions of the corresponding latent factors.[21] Therefore, using the moving average representation of the factors, the credit (resp. liquidity) proxy is a combination of past (resp. past and current) liquidity shocks, of past and current (resp. past) credit shocks and of a measurement error $\nu_{c,t}$ (resp. $\nu_{l,t}$). Formally:

$$\begin{cases} P_{c,t} & = & \pi_{c,0} + \pi_{c,1}x_{c,t} + \pi_{c,2}x_{c,t}^2 + \sigma_{\nu_c}\nu_{c,t} \\ P_{l,t} & = & \pi_{l,0} + \pi_{l,1}x_{l,t} + \pi_{l,2}x_{l,t}^2 + \sigma_{\nu_l}\nu_{l,t} \end{cases} \tag{9}$$

---

[16]This premium stems from various features of Treasury securities, such as repo specialness (see Feldhutter and Lando [2008]).

[17]This survey addresses issues such as credit standards for approving loans as well as credit terms and conditions applied to enterprises and households. The survey is addressed to senior loan officers of a representative sample of Euro area banks. The sample group participating in the survey comprises around 90 banks from all Euro area countries.

[18]The respondents can answer $++$, $+$, 0, $-$ or $--$ to that question. Our indicator is computed as the proportion of $-$ and $--$ as a ratio of total answers.

[19]nearly 75% of the total variance is explained by the first principal component.

[20]In a context of mounting fears of euro-area break-up, the President of the ECB, Mario Draghi, declared that "the ECB [was] ready to do whatever it takes to preserve the euro" at the Global Investment Conference in London, 26 July 2012.

[21]This relationship, of the same kind of the one relating the latent factors to modelled spreads, is consistent with the fact that several variables used in the computation of proxies are also homogeneous to interest rates.

Figure 2: Proxies dynamics

Credit risk proxy

Liquidity risk proxy
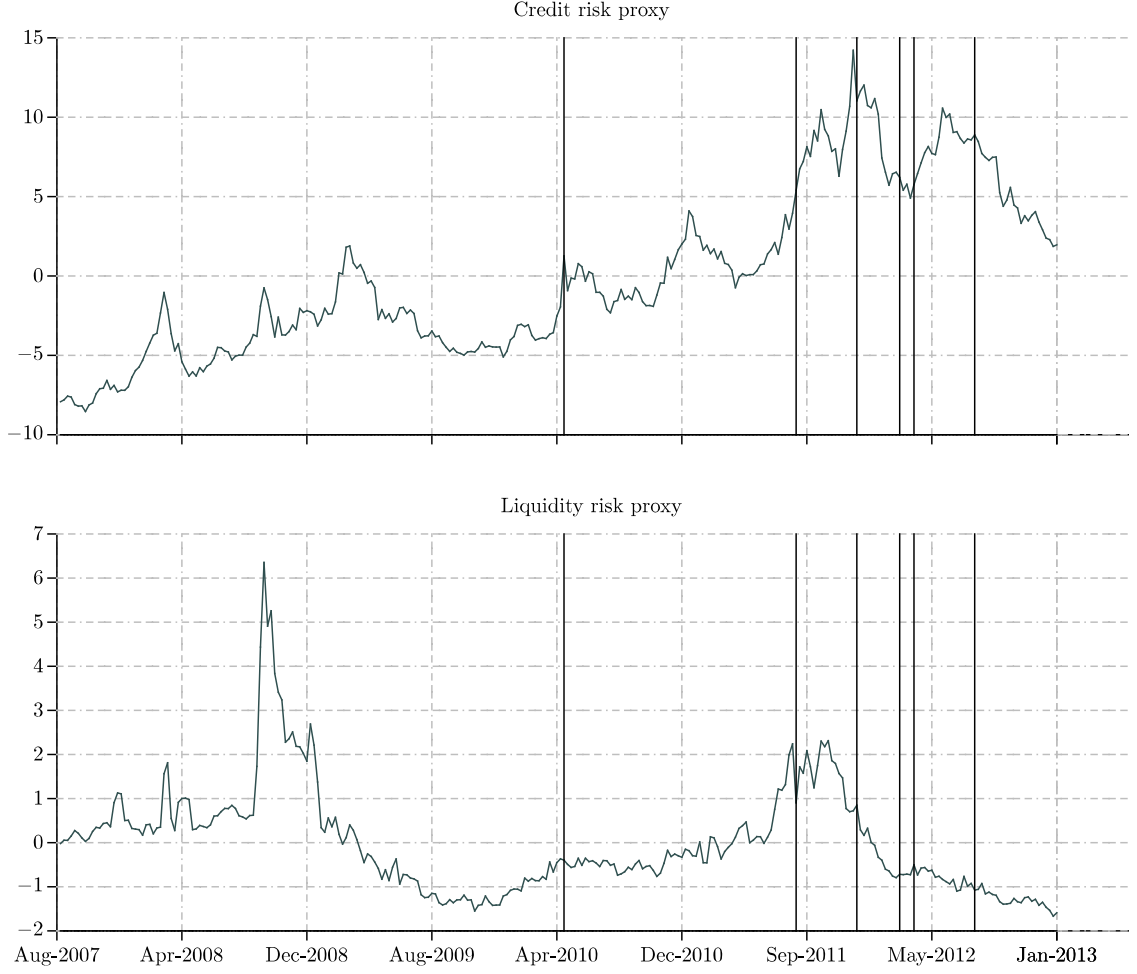
Notes: Time ranges from August 31, 2007 to January 4, 2013. Solid lines are observed proxies. The black vertical axes stand from left to right for: SMP program announcements (first two axis), VLTRO announcement and allotments (next three axis), and Mario Draghi's London speech (last axis).

where $(\nu_{c,t}, \nu_{l,t})' \sim \mathcal{IIN}(0, I_2)$ are the measurement errors on the proxies. As long as $x_{c,t}$ and $x_{l,t}$ are not instantaneously correlated, the same is true for the proxies. Moreover, as for the factors $x_{c,t}$ and $x_{l,t}$, we assume that there is no instantaneous causality between the two proxies.

The state-space representation of the model is obtained by gathering: (a) the $\mathbb{P}$-dynamics of the factors $x_{c,t}$ and $x_{l,t}$ (Equation (5)), (b) the spread formulae (Equation (7)) and (c) the proxies measurement equations (Equation (9)).

$$
\begin{aligned}
Transition: \quad & \begin{pmatrix} x_{c,t} \\ x_{l,t} \end{pmatrix} = \begin{pmatrix} \mu_c \\ \mu_l \end{pmatrix} + \begin{pmatrix} \varphi_{1,1} & \varphi_{1,2} \\ \varphi_{2,1} & \varphi_{2,2} \end{pmatrix} \begin{pmatrix} x_{c,t-1} \\ x_{l,t-1} \end{pmatrix} + \begin{pmatrix} \sigma_c & 0 \\ 0 & \sigma_l \end{pmatrix} \begin{pmatrix} \varepsilon_{c,t} \\ \varepsilon_{l,t} \end{pmatrix} \\
Measurement: \quad & \begin{cases} S_t = \theta_0 + \theta_1(x_{c,t} + x_{l,t}) + \theta_2(x_{c,t} + x_{l,t})^2 + \sigma_\eta \eta_t \\ P_{i,t} = \pi_{i,0} + \pi_{i,1} x_{i,t} + \Pi_i x_{i,t}^2 + \sigma_{\nu,i} \nu_{i,t} \qquad \forall i = \{c, l\} \end{cases}
\end{aligned}
\tag{10}
$$

where S_t is the vector of observed spreads; the vector of pricing errors η_t is composed of independent

Gaussian white noises with unit variance; vectors θ_0, θ_1, and θ_2 are composed of $\theta_{0,h}$, $\theta_{1,h}$, and $\theta_{2,h}$ for the four considered maturities. All the parameters of the $\mathbb{P}$-dynamics and μ^* and φ^* are identifiable.[22] The estimation constraints on the parameters are presented in Appendix A.2.

5.3 The Augmented Kalman Filter for QTSM estimation

We estimate the state-space model with maximum likelihood techniques accompanied with a non-linear Kalman filter. Whereas recent articles use extensively the so-called Unscented Kalman Filter (UKF, see for instance Filipovic and Trolle [2011] or Christoffersen, Dorin, Jacobs, and Karoui [2012]), we rely on the Augmented Kalman filter (AKF) of Monfort, Renne, and Roussellet [2013] fitted to quadratic measurement equations.

The AKF is based on the fact that the measurement equations are quadratic in the latent factor $X_t = (x_{c,t}, x_{l,t})'$ but affine in the stacked vector $W_t = (X_t', Vech(X_tX_t'))'$. This stacked vector W_t defines a new state-space representation, and new factor dynamics. In particular, the measurement equations can be transformed as:

$$\begin{pmatrix} S_{t,h} \\ P_{c,t} \\ P_{l,t} \end{pmatrix} = \begin{pmatrix} \theta_{0,h} \\ \pi_{c,0} \\ \pi_{l,0} \end{pmatrix} + \begin{pmatrix} \theta_{1,h} & \theta_{1,h} & \theta_{2,h} & 2\theta_{2,h} & \theta_{2,h} \\ \pi_{c,1} & 0 & \pi_{c,2} & 0 & 0 \\ 0 & \pi_{l,1} & 0 & 0 & \pi_{l,2} \end{pmatrix} W_t + \begin{pmatrix} \sigma_\eta \eta_{t,h} \\ \sigma_{\nu_c} \nu_{c,t} \\ \sigma_{\nu_l} \nu_{l,t} \end{pmatrix},$$

where $W_t = \begin{pmatrix} x_{c,t} & x_{l,t} & x_{c,t}^2 & x_{c,t}x_{l,t} & x_{l,t}^2 \end{pmatrix}'$.

Monfort, Renne, and Roussellet [2013] show that the first two moments of W_t conditional on its past values are available in exact closed-form. It allows to approximate this conditional distribution of W_t even though it has no closed-form. Once the state-space model is rewritten as a function of W_t, a standard linear Kalman-Bucy Filter can be applied for filtering and estimation purposes the conditional distribution of W_t being assumed to be Gaussian.[23]

5.4 Estimation results

We compute the estimations on weekly data from August 31, 2007 to January 4, 2013. The EURIBOR and OIS data are extracted from Bloomberg for the following maturities: 3, 6, 9, and 12 months. Table 2 reports the estimates of the parameters specifying the historical dynamics of factor $x_{c,t}$ and $x_{l,t}$. Both processes are very persistent through time, as the diagonal coefficient are close to one. Figure 3 represents the evolution of the latent factors in our sample.

Both standard errors of the residuals are significant, and give an intuition on the size of each idiosyncratic shocks in the common factor x_t. The liquidity shocks weighs more in the variance of the innovations of x_t as σ_l is largely above σ_c. Figure 3 illustrates the higher volatility of the liquidity component compared to the credit one, and it experiences a more irregular behaviour with large

---

[22] The specific case of the identifiability of σ_c and σ_l parameters is treated in appendix A.2.

[23] The parameter estimates stem from the maximization of the likelihood function. In order to avoid local maxima issues, the estimation is achieved in two steps. The *Artificial Bee Colony* stochastic algorithm (see Karaboga and Basturk [2007]) is used to find the potential maxima areas of parameters. The results are then used as starting values for a usual simplex maximization algorithm and the best estimate is selected.

Table 2: Factor parameter estimates

	intercept	$x_{c,t-1}$	$x_{l,t-1}$	$\varepsilon_{c,t}$	$\varepsilon_{l,t}$
$x_{c,t}$	$0,0004$	$0,9932^{***}$	$0,0023$	$0,1992^{***}$	0
	$(0,0368)$	$(0,0053)$	$(0,0024)$	$(0,0148)$	
$x_{l,t}$	$0,2993$	$-0,0275$	$0,9783^{***}$	0	$0,9800^{***}$
	$(0,3054)$	$(0,0546)$	$(0,0140)$		$(0,0030)$

Notes: Robust standard errors are in parentheses. Significance code: '***' for p-value < 0.01, '**' for p-value < 0.05, '*' for p-value < 0.1.

jumps. Such jumps are manifest in late 2008 and late 2011, when the Lehman collapse and the tensions on the European sovereign markets were associated with large positive shock on the liquidity factor.

Figure 3: Factor physical dynamics
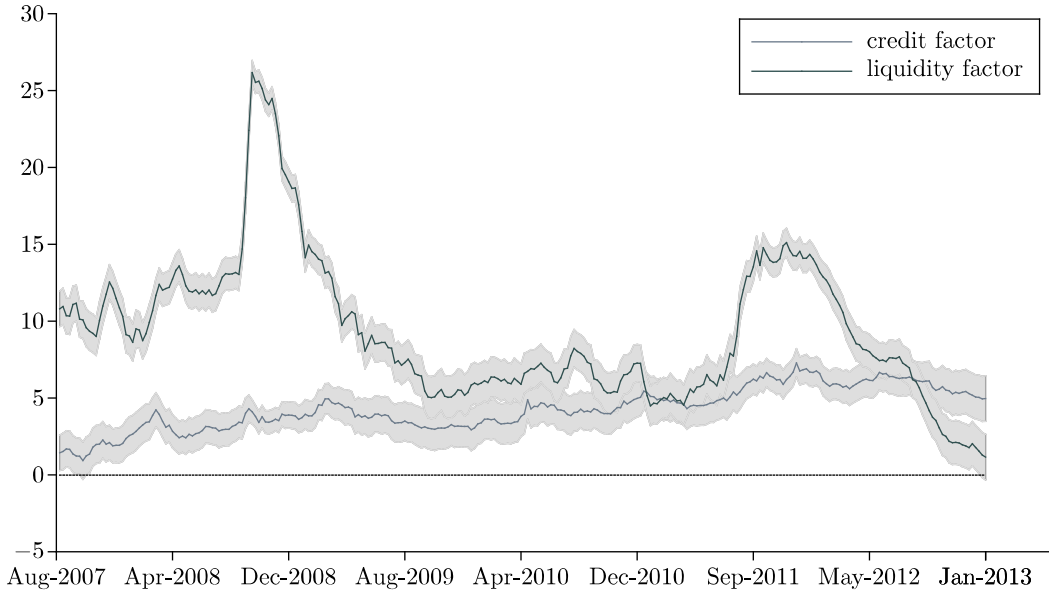
Notes: Time ranges from August 31, 2007 to January 4, 2013. The grey shaded areas are the 95% confidence intervals of the latent factors. The horizontal black line is the *argmin* of the intensity function given in Equation (3).

The remaining parameter estimates are gathered in Table 3, which shows the prevalent effect of the quadratic term in the intensity specification (Equation (3)) and in those of the proxies. These results emphasize the importance of quadratic terms in the spread modelling. The risk-neutral parameters also show a great persistence of x_t in the risk-neutral world.

Our model features remarkable fitting properties on both the observed spreads (see Table 3). In particular, the standard deviations of the pricing errors is about 10 basis points. Lastly, Table 4 reports the factor loadings associated with the spread specification for the different maturities. The loadings used in the pricing formulae are those under the $\mathbb{Q}$-measure (first four rows) whereas those under the $\mathbb{P}$ measure are used to obtain the decomposition of the spread in expected value and term premia (see Figure 4, bottom figure).

Even if the size of the spreads factor loadings are difficult to interpret, it is useful to look at the

Table 3: Risk-neutral and measurement parameter estimates

Equation	Parameter	Estimate	Parameter	Estimate	Parameter	Estimate
x_t	μ^*	$0,2676^{***}$	φ^*	$0,9973^{***}$		
		$(0,0363)$		$(0,0018)$		
$P_{c,t}$	$\pi_{c,0}$	$-8,8447^{***}$	$\pi_{c,1}$	$-0,00002$	$\pi_{c,2}$	$0,4376^{***}$
		$(0,3464)$		$(3,6923)$		$(0,0605)$
$P_{l,t}$	$\pi_{l,0}$	$-1,5411^{**}$	$\pi_{l,1}$	$0,1187^{***}$	$\pi_{l,2}$	$0,0040^{***}$
		$(0,2047)$		$(0,0225)$		$(0,0005)$
λ_t	λ_0	$0,0724$	λ_1	$0,0001$	λ_2	$0,0019^{***}$
		$(0,0840)$		$(0,0118)$		$(0,0003)$
noise	$\sigma^2_{\nu_c}$	$0,0066$	$\sigma^2_{\nu_l}$	$0,1000$	σ^2_{η}	$0,0105^{***}$
		$(0,0098)$		—		$(0,0005)$

Notes: Robust standard errors are in parentheses. Significance code: '***' for p-value < 0.01, '**' for p-value < 0.05, '*' for p-value < 0.1. The '—' sign indicates that the constraint on $\sigma^2_{\nu_l}$ is binding thus the parameter is not estimated.

Table 4: Factor loadings estimates

$\mathbb{Q}$-measure	Intercept	x_t	x_t^2			
Spread 3M	0,0938	0,0069	0,0018			
Spread 6M	0,1277	0,0128	0,0018			
Spread 9M	0,1742	0,0183	0,0017			
Spread 12M	0,2320	0,0234	0,0016			
$\mathbb{P}$-measure	intercept	$x_{c,t}$	$x_{l,t}$	$x_{c,t}^2$	$x_{l,t}^2$	$x_{c,t}x_{l,t}$
Spread 3M	0,1030	0,0054	0,0062	0,0012	0,0014	0,0013
Spread 6M	0,1349	0,0068	0,0094	0,0008	0,0012	0,0009
Spread 9M	0,1673	0,0064	0,0110	0,0006	0,0009	0,0007
Spread 12M	0,1985	0,0050	0,0115	0,0004	0,0008	0,0005

Notes: The constraints on the $\mathbb{Q}$-dynamics to identify μ^* and φ^* impose that only x_t and x_t^2 are involved in the observed spread modelling (see Equation (7)). However, such constraints are not imposed for the $\mathbb{P}$-dynamics thus $x_{c,t}$ and $x_{l,t}$ possess differentiated influences on the spread under the expectation hypothesis. Hence for each maturity, three factor loadings are needed under the $\mathbb{Q}$-measure and six are needed under the $\mathbb{P}$-measure.

derivatives of the spreads equations. Suppose that we shock the x_t factor of 1 unit. The instantaneous effect on the spreads depends on the current value of x_t and not only on the size of the shock Δx_t. These effects are approximately given by:

$$\frac{\partial S(t,h)}{\partial x_t} = \begin{cases} 0,0069 + 0,0018 \times x_t & \text{for} \quad h = 3M \\ 0,0128 + 0,0018 \times x_t & \text{for} \quad h = 6M \\ 0,0183 + 0,0017 \times x_t & \text{for} \quad h = 9M \\ 0,0234 + 0,0016 \times x_t & \text{for} \quad h = 12M \end{cases}$$

This highlights the non-linear aspect of our model. If the interbank market is already in distress, then the agents react more strongly to any modification of the underlying risks. For instance, at the peak of Lehman crisis and in January 2013 and 2 months after the OMT announcement, the values of x_t are around 31 and 9, respectively. The respective instantaneous response to a 1 standard error unit positive shock on the factor at those periods on the 3-month maturity spread would be 6,27 in 2008 and 2,31 bps in 2013 (of which 0,69 bps is constant in time).

6 Decomposing EURIBOR-OIS spreads

In this section, we derive a simple decomposition method for our QTSM specification and decipher the credit and liquidity components in EURIBOR-OIS spreads for all maturities.

6.1 The decomposition method

Rewriting the spreads equation, we get:

$$
\begin{aligned}
S(t,h) &= \theta_{0,h} + \theta_{1,h}x_t + \theta_{2,h}x_t^2 \\
&= \underbrace{\theta_{1,h}x_{c,t} + \theta_{2,h}x_{c,t}^2}_{\text{credit spread}} + \underbrace{\theta_{1,h}x_{l,t} + \theta_{2,h}x_{l,t}^2}_{\text{liquidity spread}} + \underbrace{2\theta_{2,h}x_{c,t}x_{l,t}}_{\text{interaction}} + \theta_{0,h}
\end{aligned}
\tag{11}
$$

The spreads can be separated in four different parts. Two first parts are the credit and liquidity components of the spreads. A third term that we call *interaction* represents the price effect of the joint presence of both risks in the economy. As $\theta_{2,h}$ is positive, the interaction becomes more positive as the two factors co-move, and more negative if they evolve in opposite directions. The fourth component is the intercept $\theta_{0,h}$ which is constant through time, and cannot be attributed to any of those three parts excepted arbitrarily. As a consequence, we set it apart and focus on the time-varying terms.

Our approach is more direct than Smith [2010] who sets the market price of risk to zero to construct the decomposition of spreads. With this method, she obtains a decomposition of the term premia in liquidity and credit risks. In comparison, our decomposition method makes it possible to decompose both observed spreads and the term premia. In addition, our identification scheme is more relevant as we authorize lagged Granger causality in the latent factors, stating that both credit and liquidity risks can be intertwined, but are forced to be instantaneously independent.

We are also able to decompose the spreads under the physical measure (or under the expectation hypothesis). Using the estimated $\mathbb{P}$-dynamics parameters (see Table 2), we calculate a new set of factor loadings under the expectation hypothesis $\theta_{0,h}^{\mathbb{P}}$, $\theta_{1,h}^{\mathbb{P}}$, and $\theta_{2,h}^{\mathbb{P}}$ (see Table 4). Those coefficients are now functions of $(\lambda_0, \lambda_1, \lambda_2, \mu_c, \mu_l, \varphi_{1,1}, \varphi_{1,2}, \varphi_{2,1}, \varphi_{2,2})$. Contrary to the risk-neutral parameters μ^* and φ^*, the physical parameters are less constrained and authorize $x_{c,t}$ and $x_{l,t}$ to have a differentiated impact on the spreads under the expectation hypothesis. The decomposition writes:

$$
S^{\mathbb{P}}(t,h) = \underbrace{\theta_{1,h,c}^{\mathbb{P}}x_{c,t} + \theta_{2,h,c}^{\mathbb{P}}x_{c,t}^2}_{\text{credit spread}} + \underbrace{\theta_{1,h,l}^{\mathbb{P}}x_{l,t} + \theta_{2,h,l}^{\mathbb{P}}x_{l,t}^2}_{\text{liquidity spread}} + \underbrace{2\theta_{2,h,cl}^{\mathbb{P}}x_{c,t}x_{l,t}}_{\text{interaction}} + \theta_{0,h}^{\mathbb{P}}
\tag{12}
$$

The relative shares of the spreads attributed to the credit, liquidity and interaction parts being very similar to the risk-neutral decomposition depicted on Figure 4, panel (a), we do not present the related graphs for the sake of clarity.

6.2 Decomposition results

The decompositions of 6-month maturity spread are represented on Figure 4. On average, the liquidity component accounts for most of the spread averages over the sample period representing more than 44% of the spreads for all maturities (see Table 5). The interaction represents between 15 and 25 % of the spreads, and the credit component represents about 13% of the spreads.

Table 5: Descriptive statistics of spread components

|	Credit	Liquidity	Interaction	$\theta_{0,h}$	Credit	Liquidity	Interaction
		$\mathbb{Q}$-components				*Term premia*	
	average (level in bps)						
Spread 3M	6,64	26,42	14,38	9,38	1,97	4,71	4,11
Spread 6M	9,04	31,27	13,90	12,77	4,56	9,84	6,48
Spread 9M	11,28	35,77	13,43	17,42	7,44	15,25	7,96
Spread 12M	13,36	39,93	12,99	23,20	10,37	20,64	8,83
	average (% of total spread avg)						
Spread 3M	11,68	46,50	25,31	16,51	3,46	8,28	7,23
Spread 6M	13,50	46,68	20,75	19,07	6,81	14,69	9,68
Spread 9M	14,48	45,92	17,24	22,36	9,56	19,57	10,21
Spread 12M	14,93	44,63	14,52	25,93	11,59	23,07	9,87

Interestingly, we also find a maturity-invariant percentage for the liquidity component average whereas the credit component average has an increased importance with maturity. This indicates that credit risk plays relatively more at the long end of the term-structure. Concentrating on the top panel of Figure 4 first top graph, we see however that the liquidity factor accounts for much of the high-frequency variations in the spreads, in particular during the distress period of stress in late 2008 (after the Lehman collapse) and in end 2011 (in a period of particular strain in the Euro sovereign markets). Panel a) and b) of Figure 4 emphasize the low-frequency fluctuations in the credit component, which increased almost monotonously since August 2007 before stabilizing in the summer 2012. This result is in line with the previous findings of Schwarz [2009] and Filipovic and Trolle [2011]. The interaction term which is always positive, is substantial over the sample period. For the 6-month maturity, the interaction term evolves between 0 and 40 bps over 2007-2013. However, looking at panel (b) of Figure 4, we observe that its evolution is smoother than the liquidity component.

The decomposition of the spread between the term premia and the spread under physical dynamics is also computed (bottom panel of Figure 4): the term premia component and the observed spread have very similar features, and are positively correlated. Figure 5 presents decompositions of the term structure of EURIBOR-OIS spreads at different dates. In particular, the bottom row shows the share of the modelled spreads that is accounted for by term premium. The longer the maturity, the larger this share.

6.3 The impact of unconventional monetary policy on interbank risk

Vis-à-vis the literature findings presented in the introduction, our application sheds a new light on the dynamics of the EURIBOR-OIS spreads, and includes the most recent ECB measures. Our estimation sample indeed encompasses the announcement of the *Securities Market Program*'s (SMP)

Figure 4: 6M EURIBOR - OIS spreads decomposition
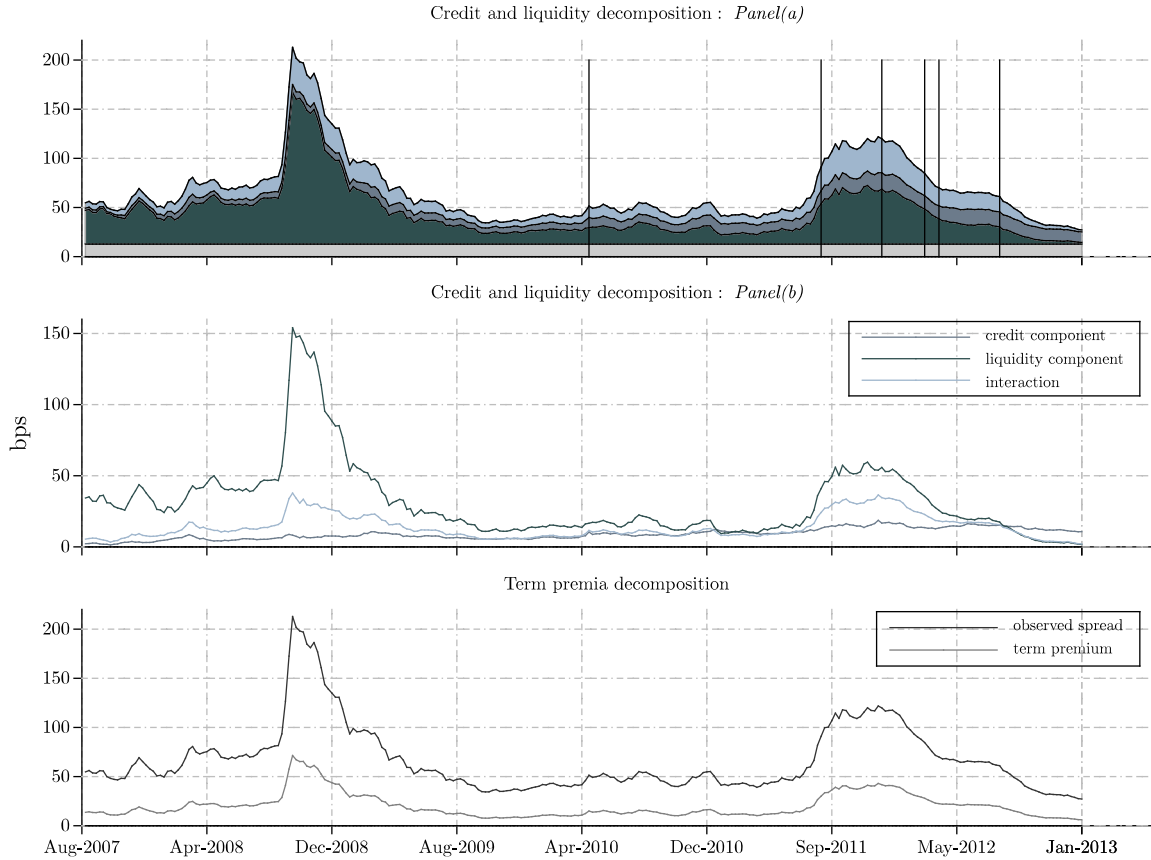
Notes: Date ranges from August 31, 2007 to January 4, 2013. Units are in basis points. *Panel(a)* represents the stacked components of the spread: light grey constant component is the intercept of the spread equation, the other legends are given on *Panel(b)* graph. *Panel(b)* represents the – not stacked – components of the spread. The graph at the bottom represents the modelled spread and its term premia. The black vertical axes stand from left to right for: SMP program announcements (first two axis), VLTRO announcement and allotments (next three axis), and Mario Draghi's London speech (last axis).

in May 2010 and August 2011, the announcement (December 2011) and allotments (February and March 2012) of the *Very Long Term Refinancing Operations*, and the announcement of the *Outright Monetary Transactions* in July 2012.[24]

Interestingly, the EURIBOR-OIS spreads have decreased continuously since the VLTRO announcement in December 2011. This drop has led many commentators (and central bankers) to claim the ECB unconventional refinancing operations were successful in alleviating interbank market tensions. In particular, according to ECB officials, the non-standard VLTRO operations addressed "only the liquidity side of the [interbank market] problem".[25] Our results seem to support this view as the liquidity component of the spreads has slowly faded away to nearly zero since the VLTRO anouncement date (see Figure 4, Panel b). A further positive effect can also be attributed to the OMT announcement through liquidity.

---

[24]As most of the market commentators, we assimilate the London's speech of Mario Draghi as the – unofficial – announcement date of the OMT.

[25]See Mario Draghi's interview with the Wall Street Journal, published on February 24, 2012, or the lecture by Peter Praet in February 20, 2012.

The same pattern can be observed on Figure 5 (first and second rows of charts). After the SMP and before the VLTRO announcement, liquidity risk still accounts for most of the term structure of interbank spreads (second column of charts). However, after the VLTRO allotments, liquidity risk represents only 10 to 30 basis points across maturities (third column) and becomes negligible for all maturities after the OMT announcement (fourth column). In comparison, looking at both Figures 4 and 5, those policy measures had virtually no impact on the credit component of the spread.

Turning to the last row of Figure 5, it appears that unconventional monetary policies were followed by decreases in both the expected component and the term premia. This result contradicts somehow the findings in Angelini, Nobili, and Picillo [2011], according to whom term premium embodies much of the spreads fluctuations. It is however difficult to compare the two results due to methodology differences: contrary to our specification, Angelini, Nobili, and Picillo [2011] blend spreads with different maturities without handling the whole term structure of spreads in a coherent no-arbitrage framework.

All in all, even if the EURIBOR-OIS spreads have not really reacted to the SMP program. Our results suggest that the recent unconventional monetary policy measures undertaken within the Eurosystem have contributed to reinforce banks liquidity positions and a stabilization of the credit risk in the Eurozone.

Figure 5: Decomposition of EURIBOR - OIS term structure
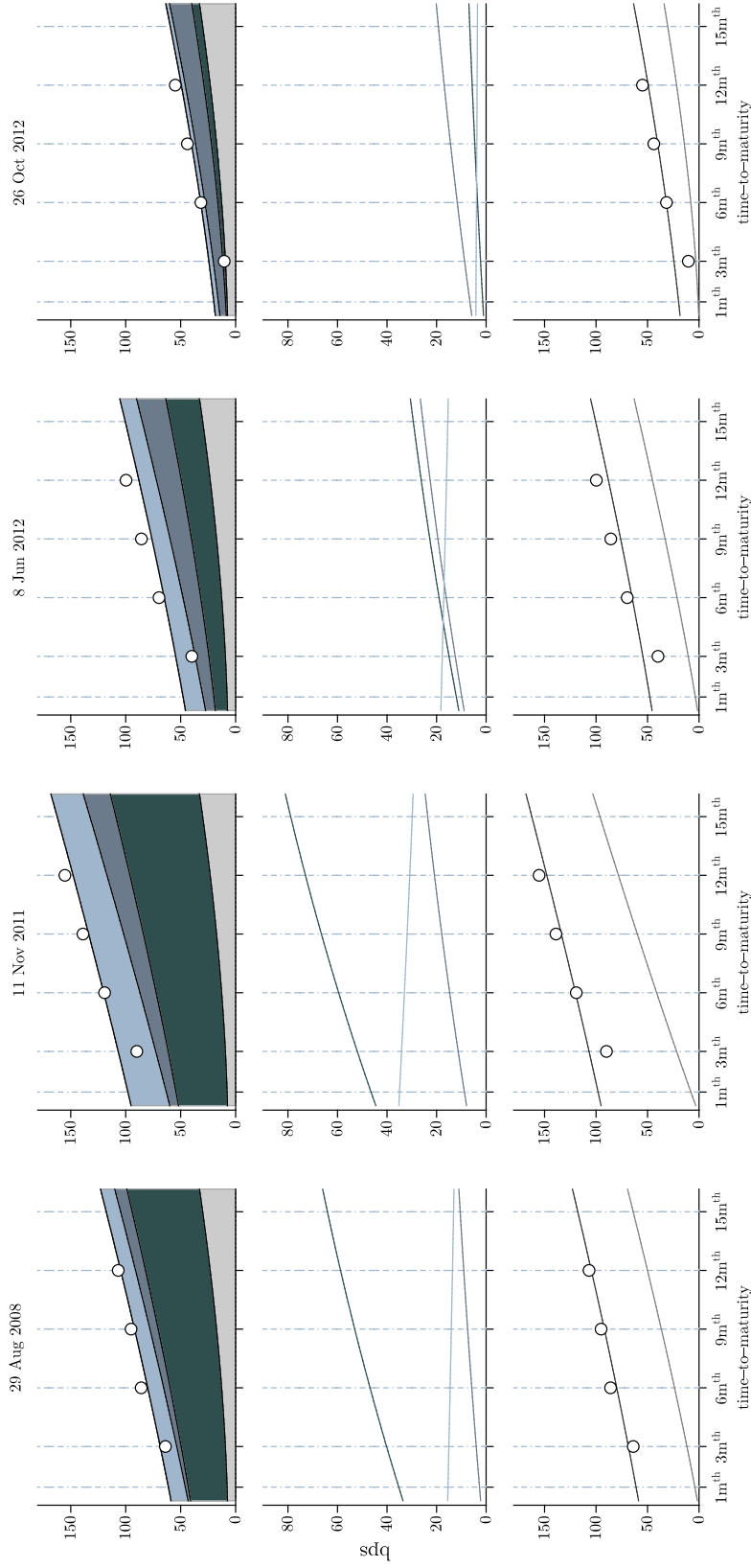
Notes: Units are in basis points. First row represents the stacked components of the term structure: lightest grey component is the intercept of the spread equation, then from dark to light are the liquidity component, credit component and interaction. Second row presents the same components (except the intercept) not stacked. The graph at the bottom represents the modelled term structure and its term premia. The white dots are the observed spreads.

Conclusion

We develop a no-arbitrage two-factor quadratic term structure model for the EURIBOR-OIS spreads across several maturities, from August 2007 to January 2013. To identify credit and liquidity components in the spreads, we introduce credit and liquidity proxies based on CDS prices, market liquidity and funding liquidity measures. Our decomposition handles potential interdependence between credit and liquidity risks and is consistent across maturities. In addition, all time-varying components of the spread can be directly interpreted with our identification scheme.

We find that the liquidity risk generates most of the variance of the spread over the estimation period. The credit risk is less volatile, but represents most of the spread level in late 2012. Our decompositions allows us to shed new light on the effects of unconventional monetary policy of the ECB on the interbank risk. We show that whereas the bond-purchase programs of 2010 and 2011 were not followed by decreases in any of the EURIBOR-OIS spread components, the VLTROs and the OMT announcements have had a substantial impact mainly, on the liquidity risk. At the end of the sample, the liquidity risk is negligible, and the remaining part of the spreads is only credit risk related.

A Appendix

A.1 Market prices of risk definition

The exponential-affine SDF between t and $t+1$, denoted $M_{t,t+1}$ is given by:

$$M_{t,t+1} = \exp\left[\Gamma_t'\begin{pmatrix} \varepsilon_{c,t} \\ \varepsilon_{l,t} \end{pmatrix} - \frac{1}{2}\Gamma_t'\Gamma_t - r_t\right], \tag{13}$$

where $\Gamma_t = \Gamma_0 + \Gamma_1(x_{c,t}\,x_{l,t})'$ corresponds to the vector of market prices of risks (MPR). The mapping between the parameters defining the historical and the risk-neutral dynamics depends on these prices of risk:

$$\begin{pmatrix} \mu_c^* \\ \mu_l^* \end{pmatrix} = \begin{pmatrix} \mu_c \\ \mu_l \end{pmatrix} + \begin{pmatrix} \sigma_c & 0 \\ 0 & \sigma_l \end{pmatrix}\Gamma_0 \tag{14}$$

$$\begin{pmatrix} \varphi_{1,1}^* & \varphi_{1,2}^* \\ \varphi_{2,1}^* & \varphi_{2,2}^* \end{pmatrix} = \begin{pmatrix} \varphi_{1,1} & \varphi_{1,2} \\ \varphi_{2,1} & \varphi_{2,2} \end{pmatrix} + \begin{pmatrix} \sigma_c & 0 \\ 0 & \sigma_l \end{pmatrix}\Gamma_1 \tag{15}$$

A.2 Identifiability and estimation constraints

Let us consider an alternative vector of factors which we call $\widetilde{X}_t = (\widetilde{x}_{c,t}, \widetilde{x}_{l,t})'$, which is an affine transformation of X_t.

$$\widetilde{X}_t = m + MX_t$$

As the proxies are respectively functions of only one component of X_t, this imposes M to be diagonal. Hence the alternative factors possess the form: $\widetilde{x}_{i,t} = M_i x_{i,t} + m_i$ for $i = \{c, l\}$. In addition, to ensure that only $\widetilde{x}_t = (\widetilde{x}_{c,t} + \widetilde{x}_{l,t})$ enters the spread formulae, we must impose $M_c = M_l$. In the end, the conditional variance of $\widetilde{x}_t$ must be equal to 1, that is

$$M_c^2\sigma_c^2 + M_l^2\sigma_l^2 = 1 \iff M_c^2(\sigma_c^2 + \sigma_l^2) = 1$$
$$\iff M_c = 1$$

Thus $M = I_2$.

For interpretation purposes, we also impose that a large proportion of the latent factors are located such that the intensity function and the proxies are monotonously increasing in the factors. That is to say:

$$\mathbb{P}\left(\frac{\partial\lambda_t}{\partial x_t}(x_{i,t}) > 0\right) = 1 - \alpha, \quad \mathbb{P}\left(\frac{\partial P_{i,t}}{\partial x_{i,t}}(x_{i,t}) > 0\right) = 1 - \alpha \quad \forall i = \{c, l\},$$

where α is typically a small number. The rationale is the following: we impose that when the latent factors are increasing, the proxies increase as well. We also want an increase in the underlying credit or liquidity risk to be translated into an increase in the spread. There must therefore be a monotonous increasing relationship between the intensity and the factors. The previous constraint implies conditions on the mean of the latent process. Denoting $\Phi = [\varphi_{i,j}]_{\substack{i=\{1,2\} \\ j=\{1,2\}}}$ and

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$$m_i = \max \left(\underset{x}{argmin} \, \lambda_t, \quad \underset{x}{argmin} \, P_{i,t} \right) \quad \forall i = \{c, l\}, \text{ we have}$$

$$\mathbb{P}\left[x_{i,t} < m_i\right] = \alpha \Longleftrightarrow \mathbb{E}(x_{i,t}) = m_i - q_{\mathcal{N}(0,1)}(\alpha)\sqrt{\mathbb{V}(x_{i,t})}$$

where $\mathbb{E}(\bullet)$ and $\mathbb{V}(\bullet)$ are the unconditional expectation and variance operators, and $q_{\mathcal{N}(0,1)}(\alpha)$ is the level α quantile of the normalized gaussian distribution. We get the following conditions on μ_c and μ_l.

$$\begin{pmatrix} \mu_c \\ \mu_l \end{pmatrix} = (I_2 - \Phi) \left[\begin{pmatrix} m_c \\ m_l \end{pmatrix} - q_{\mathcal{N}(0,1)}(\alpha)\sqrt{diag\left\{matrix_{2,2}\left[(I_4 - \Phi \otimes \Phi)^{-1} \times (\sigma_{\nu_c}^2, 0, 0, \sigma_{\nu_l}^2)'\right]\right\}} \right]$$

where the square root of the vector denotes the square root of each component. The α parameter controls the tightness of the constraint. The term in the *diag* operator is just the unconditional variance of X_t. As (μ_c, μ_l) are functions of other identified parameters, they are also identified thus $m = 0$.

In the estimation, we set $\alpha = 0.025$. We also control the accuracy of the fit of the proxies, and impose that both $\sigma_{\nu_c}^2$ and $\sigma_{\nu_l}^2$ are below 0.1.

A.3   Pricing formulas

We derive the pricing formulas in the general case when the default intensity is a quadratic functions of the factors. Three different cases are considered successively: when the variance covariance of the factor process is invertible, and when it is not.

A.3.1   Computation of factor loadings with normalized identity covariance matrix

Let us first introduce a fundamental lemma.

Lemma A.1 *If $\varepsilon_{t+1}^* \sim \mathcal{N}(0, I)$, we have*

$$\mathbb{E}_t\left[\exp(\vartheta'\varepsilon_{t+1}^* + \varepsilon_{t+1}'^* V \varepsilon_{t+1}^*)\right] = \frac{1}{|I - 2V|^{1/2}} \exp\left[\frac{1}{2}\vartheta'(I - 2V)^{-1}\vartheta\right]$$

Proof 1 *it can be shown that*

$$\forall u \in \mathbb{R}^n, \quad \int_{\mathbb{R}^n} \exp(-u'Qu + \nu'u)\,\mathrm{d}u = \frac{\pi^{n/2}}{|Q|^{1/2}} \exp\left(\frac{1/4'}{\nu} Q^{-1}\nu\right)$$

Therefore, we have:

$$
\begin{aligned}
\mathbb{E}_t\left[\exp(\vartheta'\varepsilon_{t+1}^* + \varepsilon_{t+1}'^* V \varepsilon_{t+1}^*)\right] &= \int_{\mathbb{R}^n} \exp(\vartheta'\varepsilon_{t+1}^* + \varepsilon_{t+1}'^* V \varepsilon_{t+1}^*)\frac{1}{(2\pi)^{n/2}}\exp\left(-\frac{1}{2}\varepsilon_{t+1}'^* I \varepsilon_{t+1}^*\right) \\
&= \frac{1}{(2\pi)^{n/2}}\int_{\mathbb{R}^n}\exp\left[-\varepsilon_{t+1}'^*\left(\frac{1}{2}I - V\right)\varepsilon_{t+1}^* + \vartheta'\varepsilon_{t+1}^*\right]\,\mathrm{d}u \\
&= \frac{1}{2^{n/2}\left|\frac{1}{2}I - V\right|^{1/2}}\exp\left[\frac{1}{4}\vartheta'\left(\frac{1}{2}I - V\right)^{-1}\vartheta\right] \\
&= \frac{1}{|I - 2V|^{1/2}}\exp\left[\frac{1}{2}\vartheta'(I - 2V)^{-1}\vartheta\right]
\end{aligned}
$$

∎

The EURIBOR-OIS spread at time t, with maturity h can be expressed with the price of EURIBOR loans and OIS:

$$
\begin{aligned}
D(t,h) &= \frac{D^{EUR}(t,h)}{D^{OIS}(t,h)} \\
&= \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(-\sum_{k=0}^{h-1}\lambda_{t+k+1}\right)\right] \\
&= \mathbb{E}_t^{\mathbb{Q}}\left[\exp(-\lambda_{t+1})D(t+1, h-1)\right]
\end{aligned}
$$

where $D^k(t,h)$ denotes the price of h-maturity product k at time t. We have:

$$
S(t,h) = -\frac{1}{h}\log\left(D(t,h)\right) = R^{EUR}(t,h) - R^{OIS}(t,h)
$$

We derive the factor loadings in the general case, where $X_t \in \mathbb{R}^n$. We look for closed-form formulas for some coefficients A_h, B_h, C_h, such that $\forall h$,

$$
D(t,h) = \exp(A_h + B_h' X_t + X_t' C_h X_t)
$$

where X_t represents the modelled factors in the economy. Letting $\lambda_t = \lambda_0 + \lambda_1' X_t + X_t'\lambda_2 X_t$, we get:

$$
\begin{aligned}
D(t,h) &= \mathbb{E}_t^{\mathbb{Q}}\left[\exp(-\lambda_{t+1})B(t+1, h-1)\right] \\
&= \mathbb{E}_t^{\mathbb{Q}}\left[\exp(-\lambda_0 - \lambda_1' X_{t+1} - X_{t+1}'\lambda_2 X_{t+1})B(t+1, h-1)\right] \\
&= \exp(-\lambda_0) \times \mathbb{E}_t^{\mathbb{Q}}\left[\exp(-\lambda_1' X_{t+1} - X_{t+1}'\lambda_2 X_{t+1} + A_{h-1} + B_{h-1}' X_{t+1} + X_{t+1}' C_{h-1} X_{t+1})\right] \\
&= \exp(-\lambda_0 + A_{h-1})\mathbb{E}_t^{\mathbb{Q}}\left[\exp\left((B_{h-1}' - \lambda_1')X_{t+1} + X_{t+1}'(C_{h-1} - \lambda_2)X_{t+1}\right)\right] \\
&= \exp(-\lambda_0 + A_{h-1})\mathbb{E}_t^{\mathbb{Q}}\left[\exp\left(\widetilde{B}_{h-1}' X_{t+1} + X_{t+1}'\widetilde{C}_{h-1} X_{t+1}\right)\right]
\end{aligned}
$$

with $\widetilde{B}_{h-1} \equiv B_{h-1} - \lambda_1$ and $\widetilde{C}_{h-1} \equiv C_{h-1} - \lambda_2$. The risk-neutral dynamics of X_t are normalized $VAR(1)$, that is to say $X_{t+1} = \mu^* + \Phi^* X_t + \varepsilon_{t+1}^*$ where (ε_t^*) is a white noise gaussian process with

identity covariance matrix. We relax this assumption in A.3.2.

$$
\begin{aligned}
\mathbb{E}_t^{\mathbb{Q}} & \left[\exp\left(\widetilde{B}_{h-1}' X_{t+1} + X_{t+1}' \widetilde{C}_{h-1} X_{t+1}\right)\right] \\
= & \ \mathbb{E}_t^{\mathbb{Q}} \left[\exp\left(\widetilde{B}_{h-1}'(\mu^* + \Phi^* X_t + \varepsilon_{t+1}^*) + (\mu^* + \Phi^* X_t + \varepsilon_{t+1}^*)' \widetilde{C}_{h-1}(\mu^* + \Phi^* X_t + \varepsilon_{t+1}^*)\right)\right] \\
= & \ \exp\left[\widetilde{B}_{h-1}'(\mu^* + \Phi^* X_t) + (\mu^* + \Phi^* X_t)' \widetilde{C}_{h-1}(\mu^* + \Phi^* X_t)\right] \\
& \quad \times \ \mathbb{E}_t^{\mathbb{Q}} \left[\exp\left\{\left(\widetilde{B}_{h-1}' + (\mu^* + \Phi^* X_t)'(\widetilde{C}_{h-1} + \widetilde{C}_{h-1}')\right) \varepsilon_{t+1}^* + \varepsilon_{t+1}'^* \widetilde{C}_{h-1} \varepsilon_{t+1}^*\right\}\right]
\end{aligned}
$$

where $(\widetilde{C}_{h-1} + \widetilde{C}_{h-1}') = 2\widetilde{C}_{h-1}$ as the matrix is the sum of two symmetrical matrix. Using lemma A.1, we get:

$$
\mathbb{E}_t^{\mathbb{Q}} \left[\exp\left(\widetilde{B}_{h-1}' X_{t+1} + X_{t+1}' \widetilde{C}_{h-1} X_{t+1}\right)\right] = \exp\left[\widetilde{B}_{h-1}'(\mu^* + \Phi^* X_t) + (\mu^* + \Phi^* X_t)' \widetilde{C}_{h-1}(\mu^* + \Phi^* X_t)\right]
$$
$$
\times \frac{1}{\left|I - 2\widetilde{C}_{h-1}\right|^{1/2}} \exp\left[\frac{1}{2}\left(\widetilde{B}_{h-1}' + 2(\mu^* + \Phi^* X_t)' \widetilde{C}_{h-1}\right)\{I - 2\widetilde{C}_{h-1}\}^{-1}\left(\widetilde{B}_{h-1}' + 2(\mu^* + \Phi^* X_t)' \widetilde{C}_{h-1}\right)'\right]
$$

For simplicity, we denote $K_h \equiv (I - 2\widetilde{C}_{h-1})$.

$$
\begin{aligned}
B(t,h) = & \quad \exp(A_h + B_h' X_t + X_t' C_h X_t) \\
= & \quad \exp(-\lambda_0 + A_{h-1}) \\
& \times \ \exp\left[\widetilde{B}_{h-1}' \mu^* + \widetilde{B}_{h-1}' \Phi^* X_t + \mu'^* \widetilde{C}_{h-1} \mu^* + \mu'^* \widetilde{C}_{h-1} \Phi^* X_t + X_t' \Phi'^* \widetilde{C}_{h-1} \mu^* + X_t' \Phi'^* \widetilde{C}_{h-1} \Phi^* X_t\right] \\
& \times \ \frac{1}{|K_h|^{1/2}} \exp\left[\frac{1}{2}\left(\widetilde{B}_{h-1}' + 2(\mu^* + \Phi^* X_t)' \widetilde{C}_{h-1}\right) K_h^{-1} \left(\widetilde{B}_{h-1} + 2\widetilde{C}_{h-1}(\mu^* + \Phi^* X_t)\right)\right] \\
= & \quad \exp(-\lambda_0 + A_{h-1}) \\
& \times \ \exp\left[\widetilde{B}_{h-1}' \mu^* + \widetilde{B}_{h-1}' \Phi^* X_t + \mu'^* \widetilde{C}_{h-1} \mu^* + \mu'^* \widetilde{C}_{h-1} \Phi^* X_t + X_t' \Phi'^* \widetilde{C}_{h-1} \mu^* + X_t' \Phi'^* \widetilde{C}_{h-1} \Phi^* X_t\right] \\
& \times \ \frac{1}{|K_h|^{1/2}} \exp\left[\frac{1}{2}\left(\widetilde{B}_{h-1}' + 2\mu'^* \widetilde{C}_{h-1} + 2X_t' \Phi'^* \widetilde{C}_{h-1}\right) K_h^{-1} \left(\widetilde{B}_{h-1} + 2\widetilde{C}_{h-1} \mu^* + 2\widetilde{C}_{h-1} \Phi^* X_t\right)\right]
\end{aligned}
$$

With the previous formula, the computation of the factor loadings can be achieved recursively. We therefore express the A_h, B_h, ahd C_h coefficients as a function of A_{h-1}, B_{h-1}, C_{h-1}, and the other parameters. We proceed term by term, from the quadratic component to the constant.

The term in $X_t' X_t$ in the exponential is:

$$
X_t' \Phi'^* \widetilde{C}_{h-1} \Phi^* X_t + \frac{1}{2}\left[2X_t' \Phi'^* \widetilde{C}_{h-1} K_h^{-1} \times 2\widetilde{C}_{h-1} \Phi^* X_t\right] = X_t' \left[-\Delta + \Phi'^* \widetilde{C}_{h-1} \Phi^* + 2\Phi'^* \widetilde{C}_{h-1} K_h^{-1} \widetilde{C}_{h-1} \Phi^*\right] X_t
$$

By identification we have,

$$
\begin{aligned}
C_h & = \ \Phi'^* \widetilde{C}_{h-1} \Phi^* + 2\Phi'^* \widetilde{C}_{h-1} K_h^{-1} \widetilde{C}_{h-1} \Phi^* \\
& = \ \Phi'^* \left[\widetilde{C}_{h-1} + 2\widetilde{C}_{h-1} K_h^{-1} \widetilde{C}_{h-1}\right] \Phi^* \\
& = \ \Phi'^* \widetilde{C}_{h-1} \left[I + 2K_h^{-1} \widetilde{C}_{h-1}\right] \Phi^* \\
& = \ \Phi'^* \widetilde{C}_{h-1} \left[K_h^{-1} K_h + 2K_h^{-1} \widetilde{C}_{h-1}\right] \Phi^* \\
& = \ \Phi'^* \widetilde{C}_{h-1} K_h^{-1} \left[K_h + 2\widetilde{C}_{h-1}\right] \Phi^*
\end{aligned}
$$

Noticing that $K_h + 2\widetilde{C}_{h-1} = I$, we get

$$\boxed{C_h = \Phi'^*\widetilde{C}_{h-1}K_h^{-1}\Phi^*} \tag{16}$$

The coefficient in X_t writes

$$
\begin{aligned}
& \widetilde{B}'_{h-1}\Phi^*X_t + \mu'^*\widetilde{C}_{h-1}\Phi^*X_t + X'_t\Phi'^*\widetilde{C}_{h-1}\mu^* \\
& \quad + \frac{1}{2}\left[\left(\widetilde{B}'_{h-1} + 2\mu'^*\widetilde{C}_{h-1}\right)K^{-1}\left(2\widetilde{C}_{h-1}\Phi^*X_t\right) + 2X'_t\Phi'^*\widetilde{C}_{h-1}K_h^{-1}\left(\widetilde{B}_{h-1} + 2\widetilde{C}_{h-1}\mu^*\right)\right] \\
= & \widetilde{B}'_{h-1}\Phi^*X_t + \mu'^*\widetilde{C}_{h-1}\Phi^*X_t + (\Phi'^*\widetilde{C}_{h-1}\mu^*)'X_t + (\widetilde{B}'_{h-1} + 2\mu'^*\widetilde{C}_{h-1})K_h^{-1}\widetilde{C}_{h-1}\Phi^*X_t \\
& \quad + \left[\Phi'^*\widetilde{C}_{h-1}K_h^{-1}(\widetilde{B}_{h-1} + 2\widetilde{C}_{h-1}\mu^*)\right]' X_t \\
= & \widetilde{B}'_{h-1}\Phi^*X_t + 2\mu'^*\widetilde{C}_{h-1}\Phi^*X_t + 2\left(\widetilde{B}'_{h-1} + 2\mu'^*\widetilde{C}_{h-1}\right)K_h^{-1}\widetilde{C}_{h-1}\Phi^*X_t
\end{aligned}
$$

By identification,

$$
\begin{aligned}
B'_h & = \widetilde{B}'_{h-1}\Phi^* + 2\mu'^*\widetilde{C}_{h-1}\Phi^* + 2\left(\widetilde{B}'_{h-1} + 2\mu'^*\widetilde{C}_{h-1}\right)K_h^{-1}\widetilde{C}_{h-1}\Phi^* \\
\text{so:} \quad B_h & = \Phi'^*\widetilde{B}_{h-1} + 2\Phi'^*\widetilde{C}_{h-1}\mu^* + 2\Phi'^*\widetilde{C}_{h-1}K_h^{-1}\left[\widetilde{B}_{h-1} + 2\widetilde{C}_{h-1}\mu^*\right] \\
& = \Phi'^*\left\{\widetilde{B}_{h-1} + 2\widetilde{C}_{h-1}\left[\mu^* + K_h^{-1}\left(\widetilde{B}_{h-1} + 2\widetilde{C}_{h-1}\mu^*\right)\right]\right\} \\
& = \Phi'^*\left\{\widetilde{B}_{h-1} + 2\widetilde{C}_{h-1}K_h^{-1}\left[K_h\mu^* + \widetilde{B}_{h-1} + 2\widetilde{C}_{h-1}\mu^*\right]\right\} \\
& = \Phi'^*\left\{\widetilde{B}_{h-1} + 2\widetilde{C}_{h-1}K_h^{-1}\left[\mu^* + \widetilde{B}_{h-1}\right]\right\} \\
& = \Phi'^*\left\{2\widetilde{C}_{h-1}K_h^{-1}\mu^* + (K_h + 2\widetilde{C}_{h-1})K_h^{-1}\widetilde{B}_{h-1}\right\} \\
& = \Phi'^*\left\{2\widetilde{C}_{h-1}K_h^{-1}\mu^* + K_h^{-1}\widetilde{B}_{h-1}\right\}
\end{aligned}
$$

In the end,

$$\boxed{B_h = \Phi'^*\left\{2\widetilde{C}_{h-1}K_h^{-1}\mu^* + K_h^{-1}\widetilde{B}_{h-1}\right\}} \tag{17}$$

For computation of A_h, let us first notice that $\frac{1}{|K_h|^{1/2}} = \exp\left(-\frac{1}{2}\log|K_h|\right)$. We have:

$$
\begin{aligned}
A_h &= -\lambda_0 + A_{h-1} + \widetilde{B}'_{h-1}\mu^* + \mu'^*\widetilde{C}_{h-1}\mu^* - \frac{1}{2}\log|K_h| \\
&\quad + \frac{1}{2}\left\{\left(\widetilde{B}'_{h-1} + 2\mu'^*\widetilde{C}_{h-1}\right)K_h^{-1}\left(\widetilde{B}_{h-1} + 2\widetilde{C}_{h-1}\mu^*\right)\right\} \\
&= -\lambda_0 + A_{h-1} + \left(\widetilde{B}'_{h-1} + \mu'^*\widetilde{C}_{h-1}\right)\mu^* - \frac{1}{2}\log|K_h| \\
&\quad + \frac{1}{2}\left\{\widetilde{B}'_{h-1}K_h^{-1}\widetilde{B}_{h-1} + 2\widetilde{B}'_{h-1}K_h^{-1}\widetilde{C}_{h-1}\mu^* + 2\mu'^*\widetilde{C}_{h-1}K_h^{-1}\widetilde{B}_{h-1} + 4\mu'^*\widetilde{C}_{h-1}K_h^{-1}\widetilde{C}_{h-1}\mu^*\right\} \\
&= -\lambda_0 + A_{h-1} + \widetilde{B}'_{h-1}\left(\mu^* + \frac{1}{2}K_h^{-1}\widetilde{B}_{h-1} + K_h^{-1}\widetilde{C}_{h-1}\mu^*\right) \\
&\quad + \mu'^*\widetilde{C}_{h-1}\left(\mu^* + K_h^{-1}\widetilde{B}_{h-1} + 2K_h^{-1}\widetilde{C}_{h-1}\mu^*\right) - \frac{1}{2}\log|K_h| \\
&= -\lambda_0 + A_{h-1} + \widetilde{B}'_{h-1}K_h^{-1}\left(K_h\mu^* + \frac{1}{2}\widetilde{B}_{h-1} + \widetilde{C}_{h-1}\mu^*\right) \\
&\quad + \mu'^*\widetilde{C}_{h-1}K_h^{-1}\left(K_h\mu^* + \widetilde{B}_{h-1} + 2\widetilde{C}_{h-1}\mu^*\right) - \frac{1}{2}\log|K_h| \\
&= -\lambda_0 + A_{h-1} + \widetilde{B}'_{h-1}K_h^{-1}\left[\left(I - \widetilde{C}_{h-1}\right)\mu^* + \frac{1}{2}\widetilde{B}_{h-1}\right] + \mu'^*\widetilde{C}_{h-1}K_h^{-1}\left(\mu^* + \widetilde{B}_{h-1}\right) - \frac{1}{2}\log|K_h|
\end{aligned}
$$

Then, noticing that

$$
\begin{aligned}
\widetilde{B}'_{h-1}K_h^{-1}\widetilde{C}_{h-1}\mu^* &= \left(\mu'^*\widetilde{C}_{h-1}K_h^{-1}\widetilde{B}_{h-1}\right)' \\
&= \mu'^*\widetilde{C}_{h-1}K_h^{-1}\widetilde{B}_{h-1} \quad \text{as it is a scalar.}
\end{aligned}
$$

The previous expression simplifies into:

$$
\boxed{A_h = -\delta_0 - \lambda_0 + A_{h-1} + \widetilde{B}'_{h-1}K_h^{-1}\left(\mu^* + \frac{1}{2}\widetilde{B}_{h-1}\right) + \mu'^*\widetilde{C}_{h-1}K_h^{-1}\mu^* - \frac{1}{2}\log|K_h|} \tag{18}
$$

A.3.2 Computation with general covariance of residuals

This section is an extension to the case where the variance-covariance matrix of the residuals of the VAR is not the identity matrix. Let us assume $\varepsilon_t^* \sim \mathcal{N}(0, \Sigma)$. Let us denote $T = \Sigma^{1/2}$. We know that $\varepsilon_t^* = T\eta_t$ where η_t is a normalized Gaussian white noise. Lemma A.1 becomes:

$$
\begin{aligned}
\mathbb{E}_t\left\{\exp\left(\vartheta'\varepsilon_{t+1}^* + \varepsilon_{t+1}'^* V \varepsilon_{t+1}^*\right)\right\} &= \mathbb{E}_t\left\{\exp\left(\vartheta'T\eta_{t+1} + \eta_{t+1}'TVT\eta_{t+1}\right)\right\} \\
&= \mathbb{E}_t\left\{\exp\left(\widetilde{\vartheta}'\eta_{t+1} + \eta_{t+1}'^*\widetilde{V}\eta_{t+1}^*\right)\right\} \\
&= \frac{1}{\left|I - 2\widetilde{V}\right|^{1/2}}\exp\left\{\frac{1}{2}\widetilde{\vartheta}'(I - 2\widetilde{V})^{-1}\widetilde{\vartheta}\right\}
\end{aligned}
$$

where $\widetilde{\vartheta} = T\vartheta$ and $\widetilde{V} = TVT$. The previous expression can be decomposed:

$$
\begin{aligned}
\widetilde{\vartheta}'(I - 2\widetilde{V})^{-1}\widetilde{\vartheta} &= \vartheta'T(I - 2TVT)^{-1}T\vartheta \\
\text{and} \quad T(I - 2\widetilde{V})^{-1}T &= (\Sigma^{-1} - 2V)^{-1}
\end{aligned}
$$

In addition, we also have:

$$|I - 2TVT|^{1/2} \quad = \quad |I - 2\Sigma V|^{1/2}$$

Therefore:

$$\boxed{\frac{1}{|I - 2\widetilde{V}|^{1/2}} \exp\left\{\frac{1}{2}\widetilde{\vartheta}'(I - 2\widetilde{V})^{-1}\widetilde{\vartheta}\right\} = \frac{1}{|I - 2\Sigma V|^{1/2}} \exp\left\{\frac{1}{2}\vartheta'(\Sigma^{-1} - 2V)^{-1}\vartheta\right\}} \qquad (19)$$

which represents little change compared to the case with identity variance covariance. Let us denote $\widetilde{K}_h = (\Sigma^{-1} - 2V)^{-1}$. We can use the same methodology as in the first section with very little difference: K_h is replaced with $\widetilde{K}_h$ and the factorization differs. We obtain the following factor-loadings:

$$\boxed{\begin{aligned}
A_h &= -\lambda_0 + A_{h-1} + \widetilde{B}'_{h-1}\widetilde{K}_h^{-1}\left(\frac{1}{2}\widetilde{B}_{h-1} + \Sigma^{-1}\mu^*\right) + \mu'^*\widetilde{C}_{h-1}\widetilde{K}_h^{-1}\Sigma^{-1}\mu^* - \frac{1}{2}\log\left|\widetilde{K}_h\right| - \frac{1}{2}\log|\Sigma| \\
B_h &= \Phi'^*\left\{2\widetilde{C}_{h-1}\widetilde{K}_h^{-1}\Sigma^{-1}\mu^* + \Sigma^{-1}\widetilde{K}_h^{-1}\widetilde{B}_{h-1}\right\} \\
C_h &= \Phi'^*\widetilde{C}_{h-1}\widetilde{K}_h^{-1}\Sigma^{-1}\Phi^*
\end{aligned}}$$

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