Sovereign Debt and Systemic Risk

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Abstract

Within the framework of the current crisis, it seems that an evaluation of systemic risk in the sovereign sector is an urgent matter that needs to be tackled. Thus, we propose an application of the systemic risk measures on macroeconomic data related to sovereign debt. To the best of our knowledge, the systemic risk concept was applied only to the stock market so as to measure the riskiness of financial institutions. We propose in this paper to transpose the notion of systemic risk to the sovereign debt crisis in order to determine which are the most systemically important countries in the Euro Area. A country becomes systemically important when its debt is no longer sustainable and default on repaying its debt would cause significant adverse consequences for the entire system. As our aim is to identify the contribution of each country to the system's default, we use a Sovereign Systemic Risk measure (SsRisk) based on the Marginal Expected Shortfall (MES) propose by Brownlees and Engle (2011) and on the budgetary constraint of the analyzed countries. The MES is estimated using a Dynamic Conditional Correlation model (DCC). To compute our measure, we use daily data on the Euro zone countries on the period 2000-2011 related to the government bonds yields 10Y and quaterly macroeconomic data on countries sovereign debts. The main results of our estimations allow us to perform comparisons in terms of countries riskiness within the Eurozone.

Keywords: Systemic risk, sovereign debt, debt crisis.

JEL classification: G28, H63.

1 Introduction

What would be the consequences of a potential default of Greece, Spain, Ireland or France on the ability of other European countries to finance their debt? Do European countries have an incentive to save Greece from default? These issues are crucial, but economists have no clear answer to these questions. Why? Simply because we have not observed in the recent past a sovereign default of a major European country. Thus, we cannot scientifically assess the consequences of such a situation.

However, these issues and limitations are largely similar to those we face when evaluating the systemic risk contribution of financial institutions (Acharya et al. 2010). Even if a financial institution has never experienced bankruptcy (for example, Lehman before 2008), the impact of such a potential event on the stability of the whole financial system is central to this literature. Within this framework, one of the most popular systemic risk measures proposed thus far is the Systemic Risk index (SRISK hereafter) recently proposed by Brownlees and Engle (2011). The SRISK measures the part of the total expected system capital shortfall in a crisis that is due to a particular financial institution. The firms with the largest capital shortfall are the greatest contributors to the crisis and are the institutions considered as most systemically risky. Hence, large expected capital shortage in a crisis does not only capture individual firm vulnerability, but also systemic risk. The SRISK is based on two elements: (i) the Marginal Expected Shortfall (MES) defined as the expected equity loss of a firm when the overall market declines beyond a given threshold over a given time horizon and (ii) the leverage of the firm.

Our goal is to transpose these systemic risk measures initially developed for the market risk to the sovereign debt risk. Hence, we aim to determine what are the European countries that contribute the most to the global systemic risk of the Euro Area. To the best of our knowledge, this is the first attempt to propose a systemic sovereign debt risk measure.

In this perspective, a country is systemically important when its debt is no longer sustainable and default on repaying this debt would cause significant adverse consequences for the entire system. A default is likely to appear whenever the yield is so high that it becomes impossible for the country to raise the necessary funds to repay its outstanding obligations. Correspondingly, the MES can be defined as the expected increase in a particular government bond yield when the overall (European) government bonds market is beyond a given threshold over a given time horizon, i.e. when interest rates exceed a given threshold.

Based on the government budget constraint, we derive a new concept of SsRISK (Systemic Sovereign Risk) index that depends on the MES and on the specificities of each country in the Eurozone, like the amount of public debt, its primary deficit and its growth rate. The SsRISK allows us identify systemically important countries, that is the countries that contribute the most to systemic risk in the Euro area.

The rest of the paper is structured as follows. Section 2 presents the the theoretic framework which allows us to derive our risk measure. In section 3, we describe the econometric methodology employed to compute them. In section 4, based on the two measures, we perform an analysis of systemic risk in the Euro area and determine which countries present the most risk to the system. Section 4 concludes.

2 Systemic Sovereign Risk: A Financial Approach

Our aim is to evaluate the expected financing requirements of Eurozone countries during a debt crisis which takes place at European level. On this basis, we further determine which countries contribute the most to systemic sovereign risk. The approach we use is similar to the one applied to financial institutions. As defined by Brownlees and Engle (2011), the systemic risk of a financial institution is perceived as *"its contribution to the total capital shortfall of the financial system that can be expected in a future crisis"*. The idea underlying this definition is that, when the financial system undergoes a crisis, the failure of a financial firm - due to large capital losses - imposes a negative externality not only on the financial sector, but on the real economy as well. Thus, the higher the capital shortfall, the greater the firm's contribution to systemic risk.

In this paper, we apply the same framework in order to assess the financing requirements of European countries. For a country, systemic risk is seen as a situation where the default on repaying sovereign debt would cause significant adverse consequences for the entire system. A default is likely to appear whenever the yield is so high that it becomes impossible for the country to raise the necessary funds to repay its outstanding obligations[1]. Thus, a country is systemically risky if it is likely for it to face large capital requirements at a time when the whole system is under stress.

In order to identify and measure the financing requirements of a country, we start from the standard budgetary constraint equation[2]. Let Y_{it} be the real GDP of country i at time t and let B_{it} be the real value of public debt. The government budget constraint combines the nominal interest rate r_{it}, net inflation π_{it}, net growth in real GDP g_{it} (between $t-1$ and t), and the primary deficit def_{it} in order to obtain the evolution of the government debt-GDP

[1] We suppose that the country is unable to raise funds in any other way (i.e. seigneurage, increase in taxes, reduction of public investment).

[2] In the case of financial institutions, the risk measure is derived frosm a Basel-type regulatory constraint.

ratio:

$$\frac{B_{it}}{Y_{it}} = (r_{it} - \pi_{it} - g_{it}) \frac{B_{i,t-1}}{Y_{i,t-1}} + \frac{def_{it}}{Y_{it}} + \frac{B_{i,t-1}}{Y_{i,t-1}}. \tag{1}$$

The nominal return r_{it} and the real stock of debt B_{it} in equation (1) are averages across terms to maturity[3]. The increase in the real debt stock at time t for country i can be expressed as follows:

$$B_{it} - B_{i,t-1} = [(g_{it} + 1)(1 + r_{it} - \pi_{it} - g_{it}) - 1] \times B_{i,t-1} + def_{it} \tag{2}$$

From this standard budgetary constraint, we derive the definition of the expected financing requirements[4], that we call *debt shortage*.

Definition. *The debt shortage represents the expected increase in debt of country i at date t conditional on the emergence of a European debt crisis:*

$$DS_{i,t-1} = \mathbb{E}_{t-1}\left(B_{it} - B_{i,t-1} \middle| \, Crisis\right) \tag{3}$$

The increase in debt represents a rise in the financing requirements of the country. To deal with this need of funding, the government will have to issue new debt at a yield rate imposed by the market. The concept of debt shortage is similar to that of an institution's capital shortfall used by Brownlees and Engle (2011), i.e. the capital that a financial firm would need to raise if another financial crisis developed. Considering the above definition and the result in equation (3), we obtain the following expression for the debt shortage of a country:

$$DS_{i,t-1} \;=\; \mathbb{E}_{t-1}\left([(g_{it} + 1)(1 + r_{it} - \pi_{it} - g_{it}) - 1] \times B_{i,t-1} + def_{it} \middle| \, Crisis\right). \tag{4}$$

We consider the crisis event as the situation where the whole area - the Euro area in our case - faces a difficulty to raise funds, *i.e.* high bond rates, and to finance the mutualized debt defined as the sum of individual debts of all countries under analysis. Under the framework of Brownlees and Engle (2011), the systemic event is defined as a drop of the market index below a certain threshold, over a given period of time. Two differences can be identified when trying to apply the same logic to sovereigns. First of all, as Eurobonds are not yet implemented, we have no global index for the area. To define a crisis situation, we therefore have to build a virtual index that we compute as an average of each member state interest rate weighted by its public debt. Let us define the nominal bond rate of the market r_{mt} as

[3] See Hall and Sargent (2010) for different maturity structures of debt.

[4] For a complete development of the model refer to the appendix.

the virtual rate that should be applied to the mutualized debt in order to equalizes the sum of interest paid on the N individual debts:

$$r_{mt}\sum_{i=1}^{N}B_{it}=\sum_{i=1}^{N}r_{it}B_{it} \tag{5}$$

where m stands for the market and i for the country.

At this point, we could define a European debt crisis as an increase in this yield index over a certain threshold[5]. Nevertheless, a second problem arises. The dynamics of sovereign bond yields are not stationary, therefore the index will also be integrated of order one. In this case, the probability that the yield index becomes greater than a fixed threshold increases with time. To solve this issue, we model government bond yields as random walks:

$$r_{jt} \quad = \quad r_{j,t-1}+z_{jt} \qquad with \quad j=\{i,m\} \tag{6}$$

and we define the crisis event as an *abnormal surprise* observed in the European yield index.

Definition. *The crisis event at the European level is defined as the occurrence of an unexpected major positive innovation in the European bond yield index:*

$$Crisis: z_{mt}>C \tag{7}$$

The threshold value C can be either a conditional or an unconditional value. Depending on the choice of this value, the probability of observing the systemic event is constant or time-varying. We choose C to be the historical Value-at-Risk (VaR) at a 5% confidence level. As z_{mt} is modeled as a Garch process, $Pr(z_{mt}>C)$ will be time dependant.

Having defined the conditioning event and assuming that g_{it} and def_{it} are predetermined, we can now derive the formula for the debt shortage of a country:

$$\begin{aligned} DS_{it} &= [(g_{it}+1)\left(1+\mathbb{E}_{t-1}\left(r_{it}|\,Crisis\right)-\mathbb{E}_{t-1}\left(\pi_{it}|\,Crisis\right)-g_{it}\right)-1]\times B_{i,t-1}+def_{it} \\ &= [(g_{it}+1)\left(1+\mathbb{E}_{t-1}\left(r_{i,t-1}+z_{it}|\,z_{mt}>C\right)-\mathbb{E}_{t-1}\left(\pi_{it}|\,Crisis\right)-g_{it}\right)-1]\times B_{i,t-1}+def_{it}. \end{aligned} \tag{8}$$

DS_{it} represents the capital that a country would need to raise, if the European bond market experienced a crisis. Such a measure helps us to answer questions like what would be

[5] The sign of our inequality is reversed with respect to Brownlees and Engle's approach, as in our case, systemic events are situated in the right tail of the yields distribution.

the expected financing requirements of Greece or Italy when the European market is faced with an unexpected increase in its yield index. This financing requirement depends both on the structural factors of each country - primary deficit, public debt level, inflation and growth rate - and on the spillover effects captured by the expectation term on the yields in equation (8). This term gives the expectation of the bond rate of one country, conditional on the market being taken by surprise, and corresponds to the marginal expected shortfall (MES) used in the financial approach.

A large debt shortfall in the Euro area will cause a real crisis. The countries with the highest debt shortfall are the greatest contributors to systemic risk, thus are the countries considered as most systemically risky. Hence, large expected debt shortage in a generalized debt crisis does not just capture individual country vulnerability, but also systemic risk. As Brownlees and Engle (2011), we further use this measure to compute a systemic sovereign risk index ($SsRISK$) that will help us to classify countries by their systemic importance.

Definition. *We define the systemic sovereign risk index of government i as:*

$$SsRISK_{it} = \max\left(0, DS_{it}\right) \tag{9}$$

We take the maximum between 0 and DS_{it} as we are only interested in positive financing requirements. A negative DS_{it} means either that the individual yield of a country is negatively correlated with the market yield or that the country does not need structural financing as it is characterized by high growth, low deficit or low debt level. The percentage version of the index gives us the contribution of a country to systemic risk:

$$SsRISK\%_{it} = SsRISK_{it} / \sum_{i=1}^{N} SsRISK_{it} \tag{10}$$

Hence, the SsRISK% index measures the portion of the total expected system debt shortage in a crisis that is due to government i. Our key assumption is that debt shortages of a given country impose external costs on the other countries when they occur during a period of distress for the whole system. These costs can be viewed as externalities that are particularly severe when the entire Euro area faces difficulties to issue debt. When the economy is in a downturn, the default of a government has even harsher consequences than in normal times, on both the financial and the real sectors. Thus the shortage of capital is dangerous for one country and for its bondholders, but it is dangerous for the global system (Euro area) if it occurs just when the rest of (European) countries also need funds to finance their deficits.

3 Methodology

Computing the debt shortage of one country requires data on its public debt, primary deficit and growth rate, but also requires estimating the two expectation terms in equation (8). For the expected value of the country's yield conditional on the emergence of a European debt crisis, the methodology applied makes use of a Dynamic Conditional Correlation model, whereas for the expected inflation conditional on the same systemic event, a historical approach is being used. Details on both this methodologies are provided below.

3.1 The Dynamic Approach

To estimate the dependence between bond yields of countries and the innovations of the market yield, $\mathbb{E}_{t-1}\left(r_{it}|z_{mt} > C\right)$, we start from a simple bivariate process where innovations $\{z_{it}\}$ and $\{z_{mt}\}$ are expressed as:

$$\begin{aligned} z_{it} &= \sigma_{it}\varepsilon_{it} = \sigma_{it}\left(\rho_{it}\varepsilon_{mt} + \sqrt{1-\rho_{it}^2}\xi_{it}\right) \\ z_{mt} &= \sigma_{mt}\varepsilon_{mt} \end{aligned} \tag{11}$$

with σ_{it} and σ_{mt} the volatilities of the bond yields of each country i and of the market, ρ_{it} the correlation between the market and country i. Moreover the disturbances are independent and identically distributed over time:

$$z_t = (z_{mt}, z_{it}) \sim F \tag{12}$$

so that $\mathbb{E}_{t-1}(z_t) = 0$ and $\mathbb{E}_{t-1}(z_t z_t') = H_t$. The covariance matrix is written as:

$$H_t = \begin{pmatrix} \sigma_{it} & \rho_{it}\sigma_{it}\sigma_{mt} \\ \rho_{it}\sigma_{it}\sigma_{mt} & \sigma_{mt} \end{pmatrix} \tag{13}$$

Using this framework, we compute the conditional expectation of bond yields as follows:

$$\begin{aligned} \mathrm{E}_{t-1}(r_{it} \mid z_{mt} > C) &= \mathrm{E}_{t-1}(r_{i,t-1} + z_{it} \mid z_{mt} > C) \\ &= r_{i,t-1} + \sigma_{it}\mathrm{E}_{t-1}(\varepsilon_{it} \mid \varepsilon_{mt} > \tfrac{C}{\sigma_{mt}}) \\ &= r_{i,t-1} + \sigma_{it}\mathrm{E}_{t-1}(\rho_{it}\varepsilon_{mt} + \sqrt{1-\rho_{it}^2}\xi_{it} \mid \varepsilon_{mt} > \tfrac{C}{\sigma_{mt}}) \\ &= r_{i,t-1} + \sigma_{it}\rho_{it}\mathrm{E}_{t-1}(\varepsilon_{mt} \mid \varepsilon_{mt} > \tfrac{C}{\sigma_{mt}}) + \sigma_{it}\sqrt{1-\rho_{it}^2}\mathrm{E}_{t-1}(\xi_{it} \mid \varepsilon_{mt} > \tfrac{C}{\sigma_{mt}}). \end{aligned} \tag{14}$$

This expectation is, hence, a function of the country's bond yield volatility, σ_{it}, the correlation between the yield of the country and the market yield, ρ_{it}, and the comovement in the tails of distribution. Individual volatilities and correlations are estimated with a DCC model[6], whereas the tail expectations are computed through a non parametric approach.

3.1.1 The DCC model

A bivariate DCC is used to model the volatility and the correlations between the bond yield index and the yields of each country i. The conditional covariance matrix is decomposed as follows:

$$H_t = D_t R_t D_t, \tag{15}$$

where $D_t = \begin{pmatrix} \sigma_{it} & 0 \\ 0 & \sigma_{mt} \end{pmatrix}$ represents a 2 by 2 diagonal matrix of volatilities and where $R_t = \begin{pmatrix} 1 & \rho_{imt} \\ \rho_{imt} & 1 \end{pmatrix}$ denotes the time varying correlation matrix.

As usual, a two step approach is used to estimate this model. In the first stage, a univariate Garch(1,1) is considered for each of the eleven countries and for the market index. The conditional variance of this model class is given by:

$$\sigma_{it}^2 = \omega_i + \alpha_i z_{i,t-1}^2 + \beta_i \sigma_{i,t-1}^2, \tag{16}$$

$$\sigma_{mt}^2 = \omega_m + \alpha_m z_{m,t-1}^2 + \beta_m \sigma_{m,t-1}^2, \tag{17}$$

The model parameters are estimated using the Quasi Maximum Likelihood approach (QML).

In the second stage, we express the dynamic conditional correlation matrix as:

$$R_t = diag(Q_t)^{-1/2} Q_t diag(Q_t)^{-1/2}, \tag{18}$$

where Q_t is the pseudo conditional correlation matrix of the returns standardized by their conditional standard deviation obtained previously, $\varepsilon_t^* = \frac{z_{it}}{\sigma_{it}}$. To compute Q_t, we introduce these standardized returns into a DCC(1,1) model:

$$Q_t = (1 - \alpha_C - \beta_C)S + \alpha_C \varepsilon_{t-1}^* \varepsilon_{t-1}^{*'} + \beta_C Q_{t-1}, \tag{19}$$

[6] Arch effects were found for all the series in our sample.

where ε_t^* is the vector of standardized returns, S is the unconditional covariance matrix of ε_t^* and α_C and β_C the DCC parameters to be estimated.

The conditions for the positive definiteness of H_t can be found in Engle and Sheppard (2001) and Brownlees and Engle (2011). The former also provides consistency and asymptotic normality conditions based on the work of Newey and McFadden (1994). In order to obtain robust standard errors, the covariance matrix is computed as $A^{-1}BA^{-1}T^{-1}$, where A represents the analytic hessian and B the covariance of the scores.

3.1.2 Non parametric estimation of tail expectations

In order to compute the expectation of the bond yield conditional on a crisis event, the final elements needed are tail expectations. For obtaining these values, we use a non parametric kernel estimation. The methodology applied here follows Scaillet (2005), with two notable differences. First, in our case, the conditioning with respect to past information is not necessary since we apply the formula on standardized residuals that are independent and identically distributed. Second, the sign in the conditioning event is reversed. Let

$$\Phi(x) = \int_{-\infty}^{x/h} k(u)\, du, \tag{20}$$

where $k(u)$ is the normal kernel function and h is a positive bandwidth. Then, the tail expectations are:

$$\widehat{E}_h(\varepsilon_{mt}|\varepsilon_{mt} > \kappa) = \frac{\sum_{t=1}^{T} \varepsilon_{mt} \ \left(1 - \Phi(\frac{\kappa - \varepsilon_{mt}}{h})\right)}{\sum_{t=1}^{T} \ \left(1 - \Phi(\frac{\kappa - \varepsilon_{mt}}{h})\right)}, \tag{21}$$

$$\widehat{E}_h(\xi_{mt}|\varepsilon_{mt} > \kappa) = \frac{\sum_{t=1}^{T} \xi_{mt} \ \left(1 - \Phi(\frac{\kappa - \varepsilon_{mt}}{h})\right)}{\sum_{t=1}^{T} \ \left(1 - \Phi(\frac{\kappa - \varepsilon_{mt}}{h})\right)}, \tag{22}$$

where h is the bandwidth parameter, κ is the cutoff point and $\Phi(\cdot)$ represents the normal cumulative distribution function [7]. For the determination of the bandwidth, we follow Scaillet (2005) and fix its value at $T^{-1/5}$ times the empirical standard deviation, equal to 1 in our case. The cutoff, κ, is given by C/σ_{mt}.

[7] For more details on the formulas, see appendix C.

3.2 The Historical Approach

The second expectation term concerns the value of inflation given the occurrence of the systemic event. As inflation series do not present arch effects, the dynamic estimation is not appropriate. Hence, a historical approach is preferred when computing this conditional expectation. This methodology was first proposed by Acharya et al. (2010), and reconsidered afterwards by Brownlees and Engle (2011), in their computations of the MES. The historical rolling MES, as defined by the latter, is the average loss for a firm when the market returns are lower than a certain negative threshold, C, over a given period of time. We adapt this measure to our case, using the following formula:

$$\mathrm{E}_{t-1}(\pi_{it} \mid z_{mt} > C) = \frac{\sum_{\tau=t-W}^{T} \pi_{i\tau} \mathbb{I}_{(z_{m\tau} > C)}}{\sum_{\tau=t-W}^{T} \mathbb{I}_{(z_{m\tau} > C)}}, \tag{23}$$

where W is the length of the rolling window and $\mathbb{I}$ is an indicator function.

The formulas and definitions given until now correspond to the *short-run* or *one-period ahead* MES of Brownlees and Engle (2011) and an in-sample analysis is sufficient for computing this measure.

3.3 Forecasted Expectations

A final remark needs to be made before passing to the interpretation of our results. When computing DS_{it}, we follow once more the methodology of Brownlees and Engle (2011) and replace the expectation term by its forecasted value. The forecast corresponds to the expectation, conditional to the information at time t, of the bond yield in h-periods' time given that over this period the system experiences an ongoing debt crisis. We find that this value depends on the one-period expectation in the following way[8]:

$$\mathrm{E}_t(r_{i,t+h}|z_{m,t+j} > C, \forall j = 1, ..., h) \simeq r_{it} + h * \mathrm{E}_t(z_{i,t+1}|z_{m,t+1} > C). \tag{24}$$

This formula corresponds to what Brownlees and Engle (2011) call Long Run MES (LRMES). However, a slight difference appears between the financial institutions' approach and our own. In the first case, the formula for the LRMES corresponds to a shareholder's perspective in which the main concern is the return *over* the entire forecast period. However,

[8] See the appendix for a detailed explanation.

for bond issuers, the interest switches to the value of the yield rate *at* a precise date in the future.

4 Data and Results

The empirical analysis focuses on the contribution of Eurozone countries to the systemic risk generated in the ongoing debt crisis. In the estimation of our risk measures, we use daily data on 10-year nominal sovereign bond yields. The study period starts in May, 2000 and ends in December, 2011 (closing prices)[9]. Eleven of the Eurozone countries[10] are analyzed in order to determine the evolution in time of their systemic risk contribution. To perform our computations, we also make use of a bond yield index calculated as a weighted average of bond yields of each country in the sample. The weights correspond to the public debt of each country divided by the total public debt of the eleven countries. Drawing on the theory of portfolio risk evaluation, we choose to compute the index by using constant weights. Table 1 shows these weights as of 2011, the latest year for which annual data on debt are available. Other data used in the computation of the SsRisk index are quarterly public debt, quarterly primary deficit and quarterly growth rates for each country[11].

Table 1: Country weights

Country	**Belgium**	**Germany**	**Ireland**	**Greece**	**Spain**	**France**	**Italy**	**Netherlands**	**Austria**	**Portugal**	**Finland**
Weight (2011)	4,40%	25,39%	2,06%	4,32%	8,95%	20,88%	23,18%	4,79%	2,65%	2,25%	1,13%

As we can see from Table 1, the weights of the three biggest Eurozone countries[12], Germany, France and Italy, stand for almost 70% of the index. It is therefore expected that this index be close in value to the yields applied to these three countries. In particular, the figures show that Italy accounts for almost a quarter of the Eurozone debt, whereas France comes only third in terms of debt, even if the country is bigger than Italy in terms of size (GDP). This table also suggests that the PIGS countries - Portugal, Ireland, Greece and

[9] The main source for the data is Thomson Reuters Datastream. We thank Peter Claeys for providing us with these data.

[10] The countries considered are: Germany, France, Italy, Spain, Ireland, Greece, Portugal, Belgium, Netherlands, Austria and Finland. These countries correspond to ten out of the eleven countries that initially formed the Eurozone. The eleventh country, Luxembourg, was omitted from our sample due to missing data. Greece, who joined the area in January 2001, before the introduction of notes and coins, was added to the sample.

[11] All these data were collected from Eurostat and ECB websites.

[12] As measured by their GDP in 2011.

Spain - have less than one fifth of the Eurozone public debt (*i.e.*, less than France, Italy or Germany, taken individually). Moreover, Greece represents less than 5% of the total Eurozone debt. However, in terms of sovereign bond yields, this country has the highest ones, as shown by the figures in Table 2. If we compare the minimum, maximum or average yields of all countries, the Greek yields are always the highest. The average yield for the market index is equal to 4.34%, close in value to the average of the biggest countries. There is a rather small volatility [13] in the series of yields, Greece being the most volatile in the panel.

Table 2: Descriptive statistics of the series

	Market index	**Belgium**	**Germany**	**Ireland**	**Greece**	**Spain**	**France**	**Italy**	**Netherlands**	**Austria**	**Portugal**	**Finland**
Avg. Yield	4,29	4,25	3,88	4,92	6,25	4,41	4,05	4,53	4,03	4,11	4,97	4,02
Max	6,07	5,84	5,49	13,90	38,99	6,71	5,65	7,31	5,60	5,77	13,80	5,63
Min	3,10	2,82	1,69	3,04	3,21	3,03	2,48	3,21	2,16	2,53	3,00	2,21
Vol.	0,61	0,64	0,80	1,62	4,77	0,68	0,69	0,65	0,75	0,72	1,82	0,76
Corr.	-	0,94	0,60	0,42	0,32	0,92	0,77	0,95	0,67	0,76	0,51	0,70
Skw	0,42	0,43	-0,29	2,22	3,90	0,41	0,23	0,75	-0,06	0,28	2,74	0,11
Kur	2,43	2,51	2,68	8,33	20,80	2,39	2,41	4,06	2,57	2,45	10,68	2,49

In Figure 1, we can see the bond yields for each of the eleven countries in our sample. Only the yields after January 2008 are shown, as before this date, the evolution was very similar for all countries, at around 4%. Over the last period, the ranking by countries reveals the same top 3: Greece, Portugal and Ireland. If at the beginning of the analyzed period, all countries bond yields were extremely close together, with the emergence of the debt crisis, things changed and the sovereign yields of PIIGS countries - Portugal, Ireland, Italy, Greece and Spain - soared. At the end of 2011, the bond yield applied to Greece exceeded 35%, whereas for Portugal and Ireland it was of 13.40% and 8.40%, respectively.

A certain convergence of the long-term government bond yields for the European countries was achieved in the area by 1999, at the creation of the Eurozone. This was the result of increased harmonization of monetary and fiscal policies on the path to Economic and Monetary Union (EMU). The introduction of the euro played only a secondary role in this process (see Côté and Graham (2004) for more details). Moreover, since the ‘80s, these yields didn’t cease to decrease, the drop for some countries being quite spectacular: Spain saw its yield drop from 17% to only 3.88% in January, 1999. At the creation of the Eurozone, most countries had yields situated at around 3.9%. The figure clearly shows that this convergence was interrupted by the Eurozone debt crisis in early 2009. Actually, countries responded to the financial crisis by taking strict budgetary measures and offering rescue funds to the financial sector, measures that had the downside of increasing public debt. Ireland and Por-

[13] The volatility is computed here as the standard deviation of each series.

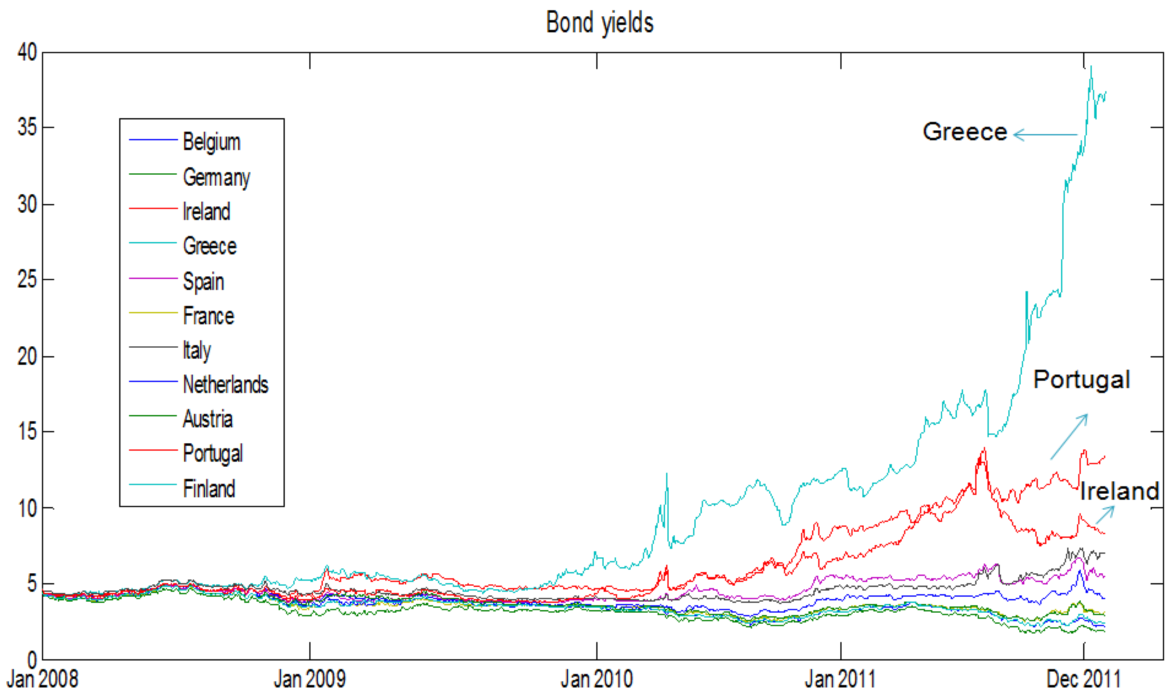


Figure 1: Bond yields by country

tugal are the countries that have experienced the largest increase in their debt during the financial crisis, whereas Greece and Italy have had historically high levels of public debt. And when investors lose confidence in a country's ability to repay its debt, high levels of debt are sanctioned by higher yields.

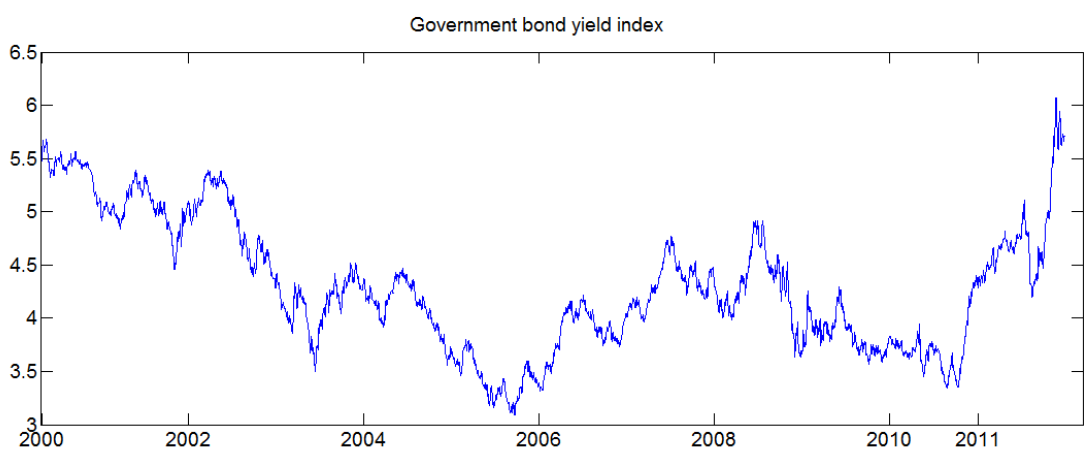


Figure 2: Government Bond Yield Index

Figure 2 provides the evolution of the index. The bond yield for the Eurozone, constructed using countries' bond yields and debt weights, is increasing since the second half of 2010. At the end of the analyzed time span, its value is much higher than the ones registered at any moment before. This is due to a sharp increase in the yields and captures the present economic turmoil.

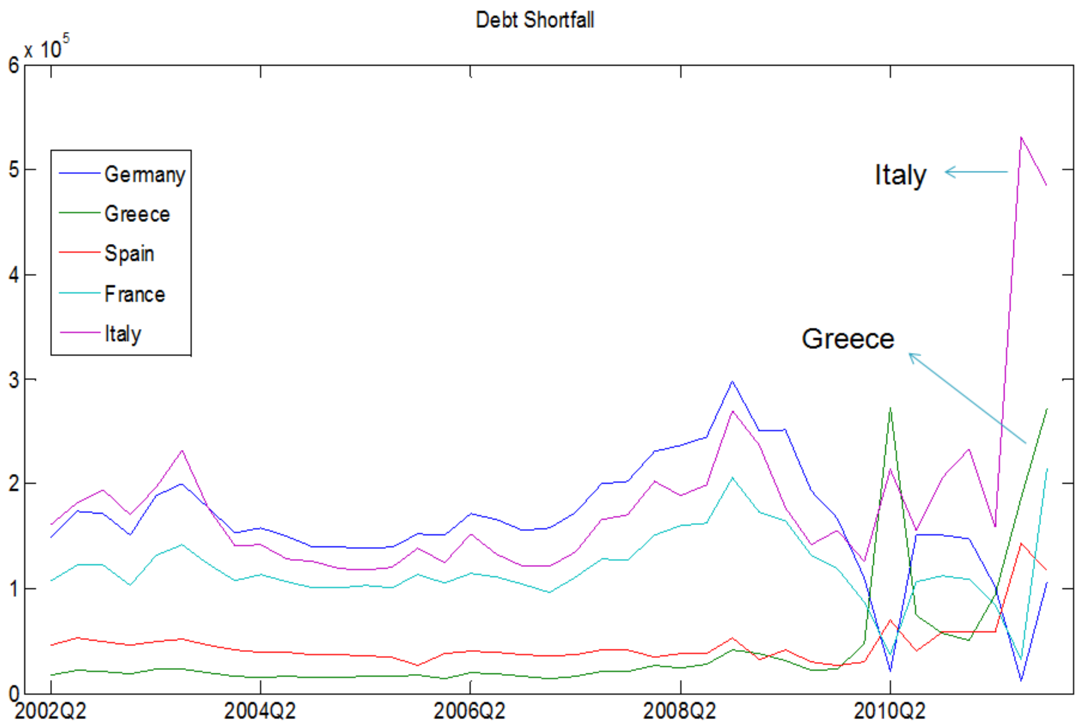


Figure 3: Debt Shortfall by country

In Figure 3, we can see that Italy is the country that, in stress periods, has the highest Debt Shortfall, thus the highest need to get financed. This makes Italy a systemically important country. Not surprisingly, Greece comes next, over the last period. Thus, even if this country accounts for less than 5% of the Eurozone debt, the figure shows that Greece is certainly systemically risky, with a sharp increase of its Debt Shortfall during 2011. Recall that during this year, financial markets became increasingly worried about a possible exit of Greece from the Eurozone, the Greek parliament voted drastic austerity measures and the European Union (EU) agreed on helping the country with several billion euros. During the same year, Greece was downgraded several times by credit rating agencies and reached the lowest investment-grade rating. All these events contributed to the abrupt increase in Greek yields. This graph reveals the importance of our measure: if we choose to analyze the data based only on the primary deficit, we get a ranking that always places Germany on the first position. Therefore, the riskiness of countries such as Italy and Greece will only be partially captured.

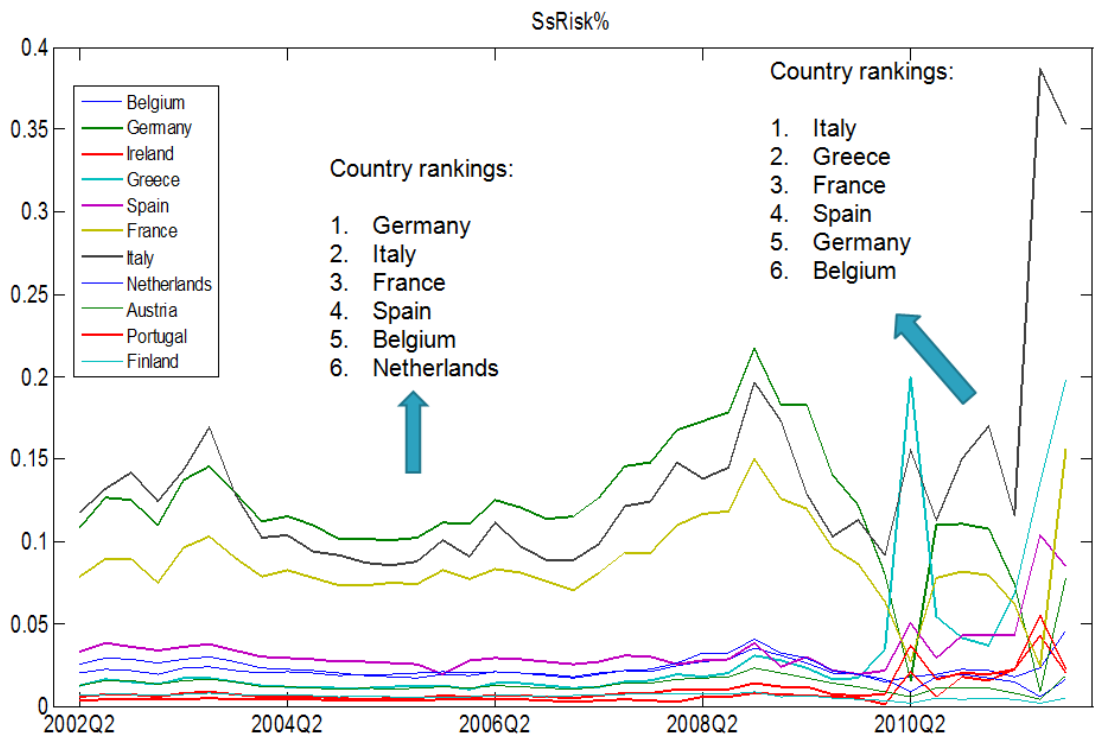


Figure 4: SsRisk Index

Figure 4 plots the SsRisk index, which has mainly the same evolution as the Debt Shortfall. Before 2010, we can remark a steady evolution of these contributions for all countries in the sample. However a quick comparison of countries' rankings, at different moments in time (*i.e.*, before and after 2010), allows us to notice that there were ranking changes. Greece, who was not in top 6 in 2008 (or before), attained the second position in 2011. Furthermore, Italy passed from the second place in 2008 to the first one at the end of the analyzed time period.In calm periods, when neither the financial markets, nor the debt yields register particular disturbances, Germany is the most systemically important country, with the highest SsRisk. In calm as well as in stress periods, France, Belgium and Spain are, without much surprise, important contributors to systemic risk.

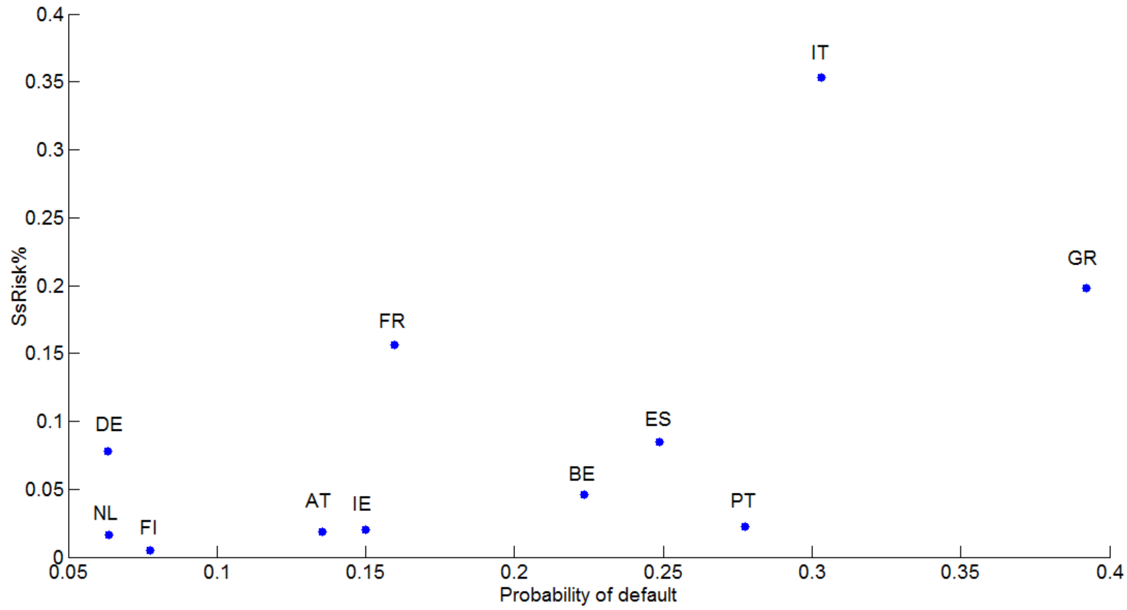


Figure 5: SsRisk and Default probabilities as of 2011 $Q4$

Figure 5 plots the SsRisk measure against the probability of default of each country, that is the probability of experiencing a systemic event. At the end of 2011, Greece and Italy were the countries with the highest probabilities of default, but also with the most important levels of SsRisk. For these countries, this underlines the danger of not only being confronted with a default, but also needing an important amount of funds in order to be rescued.

By using the same type of graph, in Figure 6 we perform a comparison between the situation before the crisis - first quarter of 2006 and after the emergence of the sovereign debt turmoil - last quarter of 2011. If we compare stress and calm periods, we notice that in 2006, countries are clustered together, having small needs of funding and low default probabilities. However in 2011, the probability of default becomes higher for all the countries (compared to 2006) and their SsRisk also gets more important, especially for countries as Italy or Greece.

When we analyze two countries in particular, for example, Greece and Germany, we can easily see in figure 7 that there is a small increase in the probability of default and SsRisk in Germany (at different moments in time) while in Greece, both the SsRisk and the probability increased sharply between 2002 and 2011, making this country systemically risky.

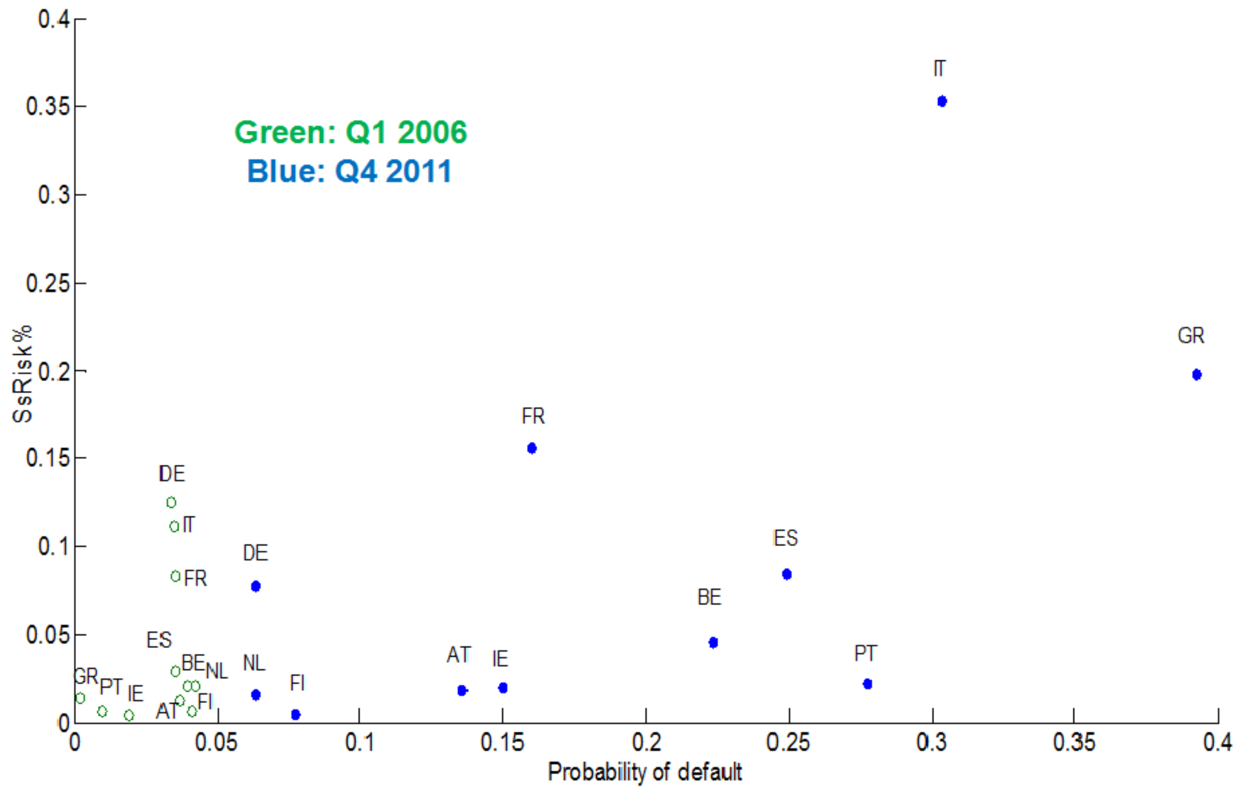
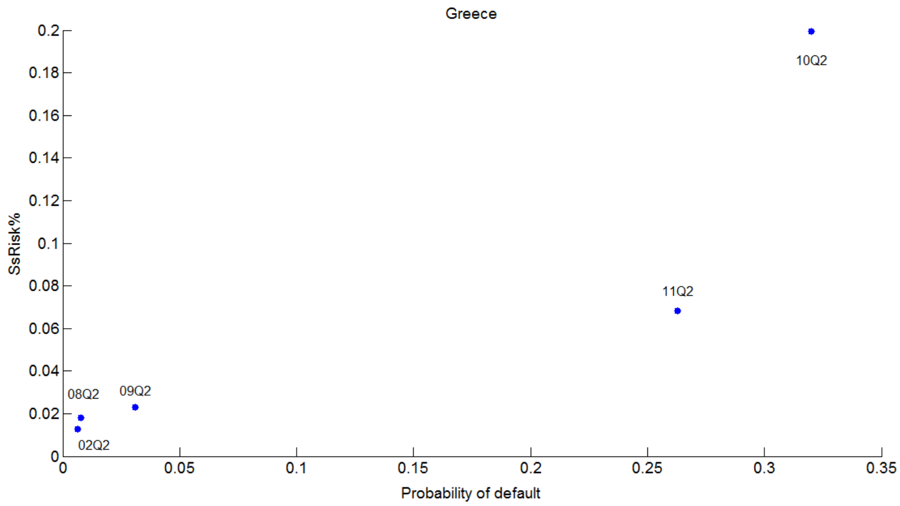


Figure 6: SsRisk - Before and during the crisis

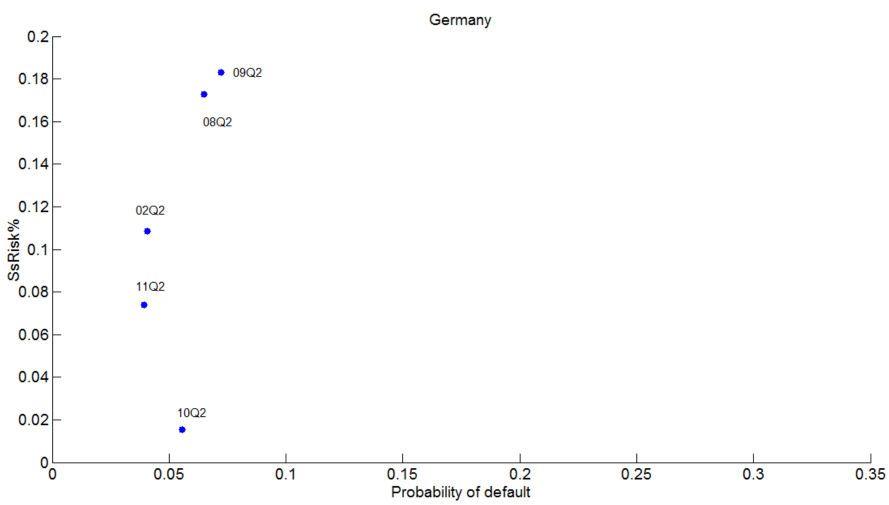
5 Policy Implications

6 Conclusions

The Marginal Expected Shortfall and the SRisk are two systemic risk measures that can successfully be applied not only to financial crises, but also to the sovereign debt crisis. Our findings confirm the fact that countries with deteriorated public finances, sanctioned by investors with high yields, have experienced in recent years an important increase in their (marginal) contribution to systemic risk. These measures give some information about which countries need more monitoring and provide a ranking of systemically important countries relative to their probability of default.



(a)



(b)

Figure 7: A country comparison

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Appendix A: Details on the calculation of MES (1)

When computing the marginal expected shortfall, we make use of a detailed expression for r_{it}. As in the main text, the bond yield of country i and the yield of the market can be written as Garch processes:

$$\begin{aligned} r_{it} &= \sigma_{it}\epsilon_{it} \\ r_{mt} &= \sigma_{mt}\epsilon_{mt} \end{aligned} \tag{25}$$

To unfold this expression, we use some standard results of the Capital Asset Pricing Model (CAPM). Mainly, the yields for country i are such that:

$$r_{it} = \beta_i r_{mt} + \mu_{it}, \tag{26}$$

with μ_{it} an error term and β_i estimated as in any linear regression:

$$\beta_i = \frac{cov(r_i, r_m)}{var(r_m)} = \frac{\sigma_{imt}}{\sigma_{mt}^2}. \tag{27}$$

Moreover, the conditional correlation between the yield of country i and the market index is given by:

$$\rho_{imt} = \frac{\sigma_{imt}}{\sigma_{it}\sigma_{mt}}, \tag{28}$$

which allows us to rewrite the equation of r_{it} as:

$$\begin{aligned} r_{it} &= \tfrac{\sigma_{imt}}{\sigma_{mt}^2}\sigma_{mt}\epsilon_{mt} + \mu_{it} \\ &= \tfrac{\sigma_{imt}}{\sigma_{mt}}\epsilon_{mt} + \mu_{it} \\ &= \tfrac{\sigma_{imt}}{\sigma_{mt}\sigma_{it}}\sigma_{it}\epsilon_{mt} + \mu_{it} \\ &= \rho_{imt}\sigma_{it}\epsilon_{mt} + \sigma_{\mu it}\xi_{it}, \end{aligned} \tag{29}$$

where μ_{it} is the residual of the linear regression and ξ_{it} is the standardized residual. Furthermore, knowing that ϵ_{mt} and ξ_{it} are orthogonal and standard normally distributed, the variance of r_{it}, $\sigma_{it}^2 = \rho_{imt}^2\sigma_{it}^2 + \sigma_{\mu it}^2$, gives us an expression for the variance of the residuals:

$$\sigma_{\mu it}^2 = \sigma_{it}^2(1 - \rho_{imt}^2), \tag{30}$$

and thus, their standard error:

$$\sigma_{\mu it} = \sigma_{it}\sqrt{(1-\rho_{imt}^2)}. \tag{31}$$

Finally, putting all these results together, we obtain the formula for the returns of asset i we are looking for:

$$\begin{aligned} r_{it} &= \rho_{imt}\sigma_{it}\epsilon_{mt} + \sigma_{it}\sqrt{(1-\rho_{imt}^2)}\xi_{it} \\ &= \sigma_{it}\underbrace{\left(\rho_{imt}\epsilon_{mt} + \sqrt{(1-\rho_{imt}^2)}\xi_{it}\right)}_{\epsilon_{it}} \end{aligned} \tag{32}$$

Appendix B: Details on the calculation of MES (2)

Departing from the expression for the expected shortfall of the Eurozone system at time t,

$$ES_{m,t-1}(C) = E_{t-1}(r_{mt}|r_{mt} > C). \tag{33}$$

we follow Scaillet (2005) and show that the first order derivative with respect the the weight associated with the i^{th} country, *i.e.* MES, is given by

$$\frac{\partial ES_{m,t-1}(C)}{\partial w_i} = E_{T-1}(r_{it}|r_m > C). \tag{34}$$

For this, we denote by $\breve{r}_{mt}$ the yield applied to the system except for the contribution of the i^{th} country, where $\breve{r}_{mt} = \sum_{j=1\ j\neq i}^{n} w_j r_{jt}$ and $r_{mt} = \breve{r}_{mt} + w_i r_{it}$. Besides, we do not restrict the threshold C to be a scalar. It is assumed to depend on the distribution of the market yield and hence on the weights and the specified probability to be in the tail of the distribution p, as in the case of the VaR, thus providing a general proof for eq. 34.

It follows that

$$\begin{aligned} ES_{m,t-1}(C) &= E_{t-1}(\breve{r}_{mt} + w_i r_{it}|\breve{r}_{mt} + w_i r_{it} > C(w_i,p)) \\ &= \frac{1}{p}\int_{-\infty}^{\infty}\left(\int_{C(w_i,p)}^{\infty}(\breve{r}_{mt} + w_i r_{it})f(\breve{r}_{mt}, r_{it})\ d\breve{r}_{mt}\right) dr_{it}, \end{aligned} \tag{35}$$

where $f(\breve{r}_{mt}, r_{it})$ stands for the joint probability density function of the two series of yields.

Consequently,

$$
\begin{aligned}
\frac{\partial ES_{m,t-1}(C)}{\partial w_i} &= \frac{1}{p}\int_{-\infty}^{\infty}\left(\int_{C(w_i,p)}^{\infty}(\breve{r}_{it})f(\breve{r}_{mt},r_{it})\;d\breve{r}_{mt}\right)dr_{it} \\
&-\frac{1}{p}\int_{-\infty}^{\infty}\left(\frac{\partial C(w_i,p)}{\partial w_i}-r_{it}\right)C(w_i,p)f\left(C(w_i,p)-w_ir_{it},r_{it}\right)dr_{it}
\end{aligned} \tag{36}
$$

However, the probability to be in the right tail of the distribution of the market yields is constant, i.e. $\Pr\left(\breve{r}_{mt}+w_ir_{it}>C\right)=p$. A direct implication of this fact is that the first order derivative of this probability is null. To put it differently, using simple calculus rules for cumulative distribution functions, it can be shown that

$$
\left(\frac{\partial C(w_i,p)}{\partial w_i}-r_{it}\right)f\left(C(w_i,p)-w_ir_{it},r_{it}\right)=0. \tag{37}
$$

Therefore, eq. 36 can be written compactly as

$$
\begin{aligned}
\frac{\partial ES_{m,t-1}(C)}{\partial w_i} &= \frac{1}{p}\int_{-\infty}^{\infty}\left(\int_{C(w_i,p)}^{\infty}(\breve{r}_{it})f(\breve{r}_{mt},r_{it})\;d\breve{r}_{mt}\right)dr_{it} \\
&= E_{t-1}(r_{it}|\breve{r}_{mt}+w_ir_{it}>C(w_i,p)) \\
&= E_{t-1}(r_{it}|r_{mt}>C),
\end{aligned} \tag{38}
$$

which completes the proof.

Appendix C: Tail Expectations

We show that the tail expectations $E_{t-1}(\varepsilon_{mt}|\varepsilon_{mt}>C/\sigma_{mt})$ and $E_{t-1}(\xi_{it}|\varepsilon_{mt}>C/\sigma_{mt})$ can be easily estimated in a non-parametric kernel framework by elaborating on Scaillet (2005).

For ease of notation, let us note the systemic risk event C/σ_{mt} by κ. We first consider the tail expectation on market yields $E_{t-1}(\varepsilon_{mt}|\varepsilon_{mt}>C/\sigma_{mt})$, which becomes

$$
E_{t-1}(\varepsilon_{mt}|\varepsilon_{mt}>\kappa). \tag{39}
$$

Using the definition of the conditional mean, we rewrite 39 as a function of the probability density function f

$$
\begin{aligned}
E_{t-1}(\varepsilon_{mt}|\varepsilon_{mt} > \kappa) &= \int\limits_{\kappa}^{\infty} \varepsilon_{mt} f(u|u > \kappa)\ du \\
&= \int\limits_{-\infty}^{\infty} \varepsilon_{mt} f(u|u > \kappa)\ du - \int\limits_{-\infty}^{\kappa} \varepsilon_{mt} f(u|u > \kappa)\ du,
\end{aligned} \tag{40}
$$

where the conditional density $f(u|u > \kappa)$ can be stated as

$$
\frac{f(u)}{Pr(u > \kappa)}. \tag{41}
$$

To complete the proof, we must compute the numerator and denominator in 41. For this, we first consider the standard kernel density estimator of the density f at point u given by

$$
\hat{f}(u) = \frac{1}{Th} \sum_{1}^{T} \phi(\frac{u - \varepsilon_{mt}}{h}),
$$

where h stands for the bandwidth parameter, and T is the sample size (Silverman, 1986, Wand and Jones, 1995, Simonoff, 1996). Second, the probability to be in the tail of the distribution can be defined as the integral of the probability density function over the domain of definition of the variable u, *i.e.* $p = Pr(u > \kappa) = \int\limits_{\kappa}^{\infty} f(u)\ du$. Consequently, by replacing $\hat{f}(u)$ with the kernel estimator, we obtain

$$
\hat{p} = \frac{1}{Th} \sum_{t=1}^{T} \left(1 - \Phi(\frac{\kappa - \varepsilon_{mt}}{h})\right).
$$

The expectation in 39 takes hence the form

$$
\hat{E}_{t-1}(\varepsilon_{mt}|\varepsilon_{mt} > \kappa) = \frac{\sum\limits_{t=1}^{T} \varepsilon_{mt} \left(1 - \Phi(\frac{\kappa - \varepsilon_{mt}}{h})\right)}{\sum\limits_{t=1}^{T} \left(1 - \Phi(\frac{\kappa - \varepsilon_{mt}}{h})\right)}. \tag{42}
$$

Similarly, it can be shown that

$$\hat{E}_{t-1}(\xi_{it}|\varepsilon_{mt} > \kappa) = \frac{\sum_{t=1}^{T} \xi_{it} \left(1 - \Phi(\frac{\kappa - \varepsilon_{mt}}{h})\right)}{\sum_{t=1}^{T} \left(1 - \Phi(\frac{\kappa - \varepsilon_{mt}}{h})\right)}. \tag{43}$$