Interest Rate Spreads and Cross-Border Banking Flows in the EMU: a DSGE Analysis

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Abstract

Over the last 15 years, international finance integration in Europe has been driven by the bank-based nature of finance of EMU participating countries, through an increase in cross border banking activities. In this paper using bayesian techniques, we address two main questions: Does it make a particular difference to have an international integration in terms of bank flows instead of bonds. Once international bank loan flows are considered, should the monetary rule be extended to take into account interest rate spreads and the loan distribution? First we find that international bank loans have promoted business cycles synchronization between countries and have increased the persistence of current account disequilibrium in the monetary union. Second, contrasting the standard taylor rule with a spread augmented taylor rule, we find that the main benefits are obtained if the economy is negatively affected by financial shocks, as the extended rule provides an attenuation mechanism in the international transmission of this kind of shocks.

Finally we study the role of cross border bank lending in the financial crisis by performing a historical decomposition of the system responses estimated with a sub-sample corresponding to the post 2007 period. We found that, the financial crisis has induced an increased contribution of financial shocks in the explanation of macroeconomic variables. The structure of our model is also helpful to underline some major evolutions in the increasing role of foreign shocks as the driving forces of national variables after the financial crisis, as well as an increased heterogeneity in the determination of investment fluctuations between Germany and France.

Keywords: Credit Spreads, Banking Globalization, Financial Accelerator, Monetary Union, DSGE models, Bayesian Estimation

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1 Introduction

This paper seeks to understand the consequences of cross border bank lending flows in shaping the transmission of asymmetric shocks and the impact of the monetary policy in a monetary union when banks provide the main funds for firms. We build a two country DSGE model with a banking system that provides cross border lending facilities. We take this model to the data by using bayesian econometrics. We more specifically estimate the main parameters on the two major economies of the eurozone, namely Germany and France.

The launching of the euro in 1999 eliminated currency risk and provided a further push for financial integration. Over the last 15 years, part of this integration has been driven by the bank-based nature of finance in Europe, through an increase in cross border banking activities. This phenomenon can be measured along various complementary dimensions such as increased FDI in bank activities, the diversification of bank assets and liabilities between countries, the access of local banks to international financial sources or the increase of banks’ lending via foreign branches and direct cross-border lending,... Among the various consequences of the increase in the cross border activities of the banking sector in Europe this paper focuses more specifically to the increase in cross border loans between european courtries. As underlined in figure 1, most of the increase in international bank flows is for members of the UEM. Figure 1a reports evidence on the latter phenomenon, as cross border loans within the euro area has been multiplied by 3 in 9 years, from 750 billions of euros in 1999 up to more than 2,100 billion euros in 2008, before the diffusion of the recent financial crisis in Europe. As clearly shown, the total amount of cross border loans has decreased by one third between 2008 and 2012, thus introducing a potential contractionary mechanism in the eurozone, through the allocation of loans in the monetary union.

The critical role of cross border loans flows must be assessed by taking into account the key role of banks in providing the main funding source for household and firms in the euro area. As reported by figure 1b, the financing structure of firms in the euro area makes investment growth sensitive to an increase in the spread between the private credit rate and the refinancing rate of the ECB. Thus interest rate spread is is affected in countries by the possibility to have access to alternative lending conditions. By so the variability of the cross border lending may also be considered as a factor that affects the heterogenous diffusion of a common monetary policy in the euro area. An increase in cross border bank flows may play a key role in the diffusion of monetary policy: by providing a better allocation of loans, it can lead to a convergence of interest rate spreads in the monetary union and thus contribute to a more homogeneous effect of monetary policy on investment. Inversely, a disruption in cross border bank flows should reduce competition in the provision of loans and in turn this may increase the interest rate spread and thus make the decision of the european central bank less operative in some countries.

Our model introduces two original features with regard to the existing literature. First, we allow for cross border lending flows in a two country model. To our knowledge, this dimension has not yet been analysed in the literature. Indeed, the existing macroeconomic literature on the european union banking can be separated in two strands: on the one hand one country models (Gerali et al., 2010), on the other hand two country model with a financial sector, that ignores the possibility of cross border funds (Faia, 2007; Kollmann et al., 2011). In the first situation, the model assumes...
complete banking integration and by so, assumes that all countries are impacted in the same way by the union wide monetary policy. On the other hand model such as assume that countries only trade financial assets, and neglect the specific effect of the banking industry in the international adjustment between countries. In this paper we take a middle in the road solution as, we find that in the german/french case, around 18% of the country can borrow from foreing country in the german/french situation.

Second, we introduce a new way of founding the accelerator phenomenon, as this plays a key role in the interaction between the financial and the real sector. The recent financial crisis has triggered a series of paper focussing on the diffusion of financial shocks in DSGE model competed to encopass the key role of banking in financing activity and the interaction between the real and the financial sectors and the consequences of lending decisions through a renewed formalization of the accelerator phenomenon (Bernanke et al., 1999, here after BGG). In our setting, bonds are mainly used, as in the intertemporal macroeconomics litterature, to allow households to smooth intertemporal consumption and countries to finance current account deﬁcit. In contrast we assume that bank loans are used by entrepreneurs to ﬁnance risky investment in complement of their net wealth. This key role of bank finance gives rise a phenomenon of financial accelerator. We assume that the asymmetry comes from the heterogeneity in the investment projects. We model heterogeneity via the pareto distribution, that is commonly used in other branch of the economic literature. This provides some interesting features with regard to the current practice initially introduced by BGG\footnote{BGG and the financial accelerator litterature modelize investment projects heterogeneity via a log normal distribution. This probability distribution is not log-linearizable and provides a non linear steady state with multiple equilibria.}: microfoundation of the accelerator effect that allows for an easier estimation of the financial ampliﬁcation, linearization, endogenous value of entrepreneurs exit.

We use this framework to investigate two speciﬁc questions.
First, we evaluate the consequences of introducing cross border lending in the transmission of asymmetric shocks. Once the model has been estimated, we evaluate its propagation mechanism under alternative scenarios. We proceed in two steps: first we evaluate the consequences of cross border lending in the transmission of shocks. In a second step we evaluate the benefits of an extended Taylor rule in a situation of cross border lending. Under a standard Taylor rule, we find that international bank flows thus do not increase the aggregate activity in the monetary union, they just promote more business cycle synchronization and mitigate the delaterious effects of financial distress. One must also note the increased persistence of shocks on the current account coming from cross border interest payments. Under an extended rule, the central bank responds negatively to an increase in the spread between the rentability of capital and the cost of borrowing. In this situation, by lowering the interbank interest rate it aims at reducing the interest rate on loans to promote borrowing at a lower interest rate, which is supposed to reduce the risks of investment by lowering the probability of bankruptcy associated to high interest payments. We find that the main benefits are obtained if the economy is affected by financial shocks, as the extended rule provides an attenuation mechanism in the international transmission of this kind of shocks. In particular, we find that this extended rule promotes a better synchronization in the financial structure of firms. This feature may be interesting since at the monetary union level, monetary policy can only target union wide variables. Introducing interest rate spreads may provide a simple solution to limit to a few countries the consequences of defavourable financial shocks, amplifying the business cycles of these countries, while preserving the economic health of the other members of the monetary union.

The second question we address in this paper is related to the role of cross border bank lending in the transmission of the financial crisis of 2007. We perform a historical decomposition of the dynamics of the four main variable of interest (namely activity, investment, loan supply and the interest rate paid by borrowers), by contrasting the whole time period (2003Q1-2012Q2) and a sub sample corresponding to the post 2007 period. The aim of the exercise is to investigate how the financial crisis of 2007 has affected the driving forces of activity, investment and loan supplies. We first begin with a general evaluation of the contribution of the shocks over the whole sample period, while reducing the time period to 2007Q3-2012Q3 in a second step. We find that the financial crisis has induced an increased contribution of financial shocks in the explanation of the evolution of the main macroeconomic variables as well as an increased in the sensitivity of output fluctuations to the interbank interest rate. The structure of our model is also helpful to underline some major evolutions in the increasing role of foreign shocks as the driving forces of national variables after the financial crisis, as well as an increased heterogeneity in the determination of investment fluctuations between Germany and France.

The rest of the paper is organised as follows, section 2 presents the model, section 3 presents the data and the econometric method, section 4 uses bayesian IRFs to evaluate the consequences of cross border bank lending and the benefits associated to the extension of the Taylor rule to encompass financial variables. the two cases with and without cross border flows. Finally, section 5 investigates how the financial crisis of 2007 has affected the driving forces of activity, investment and credit supply using a historical decomposition of these variables’ dynamics.
2 A monetary union with cross border bank lending

This first section introduces a two country DSGE model with a set of real and nominal frictions as CEE (Christiano et al., 2005) and SW (Smets & Wouters, 2007). These model features are deemed necessary to replicate the dynamics of consumption and investment, respectively. We include habit formation and investment adjustment costs / variable capital utilization. The two countries have the same size and share a common currency. Each country $i \in \{ h, f \}$ (where $h$ is for home and $f$ for foreign) is populated by consumers, intermediate and final producers, entrepreneurs, capitalists and a banking system. Regarding the conduct of macroeconomic policy, we assume national fiscal authorities and a common central bank.

2.1 Households

In each economy there is a continuum of identical households who consume, save and work in intermediate firms. The total number of households is normalised to 1. The representative household $j \in [0, 1]$ maximises the welfare index,

$$\max_{\{C_{i,t}(j), H_{i,t}(j), B_{i,t}(j)\}} \sum_{\tau = 0}^{\infty} \beta^{\tau+t} e^{\alpha^{\phi}_{t+i}} E_t \left\{ \frac{(C_{i,t+\tau}(j) - h_i^c C_{i,t-1+\tau}(j))^{1-\sigma_i^c}}{1-\sigma_i^c} - \chi_i H_{i,t+\tau}(j)\frac{1+\sigma_i^e}{1+\sigma_i^e} \right\},$$

subject to,

$$\frac{W_{i,t}}{P_{i,t}^c} H_{i,t}(j) + R_{i,t} B_{i,t}(j) + \frac{\Pi_{i,t}^y(j)}{P_{i,t}^c} + \frac{\Pi_{i,t}^B(j)}{P_{i,t}^c} = C_{i,t}(j) + \frac{B_{i,t+1}(j)}{P_{i,t}^c} + \frac{T_{i,t}(j)}{P_{i,t}^c} + \frac{P_{i,t}^L}{P_{i,t}^c} A{C_{i,t}^B(j)}$$

Here, $C_{i,t}(j)$ is the consumption index, $h_i^c \in [0; 1]$ is a parameter that accounts for consumption habits, $H_{i,t}(j)$ is labour effort, $\epsilon_{i,t}^c$ is an exogenous AR(1) shock to household preferences. The income of the representative household is made of labour income (with the nominal wage $W_{i,t}$) and the consumption price index $P_{i,t}^c$, interest payments for bond holdings, (where $B_{i,t}(j)$ stands for the bonds subscribed in period $(t-1)$ and $R_{i,t}$ is the gross nominal rate of interest between period $t-1$ and period $t$), and earnings from shareholdings (where $\Pi_{i,t}^y(j)$ and $\Pi_{i,t}^B(j)$ are the nominal amount of dividends he receives from final good producers and banks). The representative household spend this income in consumption, bond subscription and taxe payments (for a nominal amount of $T_{i,t}(j)$). Finally, we assume that the household has to pay quadratic adjustment costs to buy new bonds, according to the function, $AC_{i,t}^B(j) = \frac{X^B}{2} (B_{i,t+1}(j) - B_{i}(j))^2$, where $B_{i}(j)$ is the steady state level of bonds.

The first order conditions that solve this problem can be summarized with a Euler bond condition,

$$\frac{\beta R_{i,t+1}}{1+X^B(B_{i,t+1}(j) - B_{i}(j))} = E_t \left\{ \frac{e^{\phi_{i,t+1}} P_{i,t+1}^c}{e^{\phi_{i,t+1}} P_{i,t}^c} \left( \frac{(C_{i,t+1}(j) - h_i^c C_{i,t}(j))^{\sigma_i^c}}{(C_{i,t}(j) - h_i^c C_{i,t-1}(j))^{\sigma_i^c}} \right) \right\},$$

and a labour supply function,

$$\frac{W_{i,t}}{P_{i,t}^c} = \chi_i H_{i,t}(j)^{\sigma_i^e} (C_{i,t}(j) - h_i^c C_{i,t-1}(j))^{\sigma_i^e}$$
The consumption basket of the representative household and the consumption price index of country \(i\) are,

\[
C_{i,t} = \left( (1 - \alpha^C_i) \right)^{1/\mu} C_{hi,t}(j) \left( (1 - \alpha^C_i) \right)^{1/\mu} C_{fi,t}(j) \left( \frac{1}{1 - \alpha^C_i} \right),
\]

\[
P_{c,i,t} = \left( (1 - \alpha^C_i) \right) P_{hi,t}^{1-\mu} + \alpha^C_i P_{fi,t}^{1-\mu} \left( \frac{1}{1 - \alpha^C_i} \right).
\]

In these expressions, parameter \(\mu\) represents the elasticity of substitution between home and foreign goods and \(\alpha^C_i\) is the degree of openness of the economy. In this model, we assume home bias in consumption, so that \(\alpha^C_i < \frac{1}{2}\). The consumption index and the price index of home and foreign goods are respectively,

\[
C_{hi,t}(j) = \left( \int_0^1 C_{hi,t}(z, j) \frac{dz}{dz} \right)^{1-\epsilon}, \quad P_{hi,t} = \left( \int_0^1 P_{hi,t}(z) \frac{dz}{dz} \right)^{1-\epsilon},
\]

\[
C_{fi,t}(j) = \left( \int_0^1 C_{fi,t}(z, j) \frac{dz}{dz} \right)^{1-\epsilon}, \quad P_{fi,t} = \left( \int_0^1 P_{fi,t}(z) \frac{dz}{dz} \right)^{1-\epsilon}.
\]

Thus, we get the variety demand for final good \(z\),

\[
C_{h,i,t}(z, j) = \alpha^C_i \left( \frac{P_{hi,t}(z)}{P_{hi,t}} \right)^{1-\epsilon} \left( \frac{P_{hi,t}}{P_{fi,t}} \right)^{1-\epsilon} C_{i,t}(j),
\]

\[
C_{f,i,t}(z, j) = (1 - \alpha^C_i) \left( \frac{P_{fi,t}(z)}{P_{fi,t}} \right)^{1-\epsilon} \left( \frac{P_{fi,t}}{P_{fi,t}} \right)^{1-\epsilon} C_{i,t}(j).
\]

### 2.2 The intermediate production sector

This sector is made of three types of agents: intermediate firms, entrepreneurs and capital suppliers. The interaction between the three agents can be presented in the following way: the intermediate firm combines labour and capital to produce intermediate goods. The intermediate goods serve as an input for final firms. To produce, the intermediate firm hires labour from households and rents capital from capital suppliers. Thus, the individual intermediate firm determines the quantity of capital and labour demand. There is a close connection between an intermediate firm and an entrepreneur: to rent capital the intermediate firm asks the entrepreneur (which can be considered as the landlord of the intermediate firm) to pay for the amount that is required. To finance this renting, the entrepreneur uses personal funds (i.e., his net wealth) and may borrow the rest from the banking sector. Thus in our setting, the entrepreneur determines the amount of loans demanded in the economy, in close connection with the amount of investment. Finally, the capital supplier rents the capital stock to the intermediate firm and determines the price of the capital renting.

#### The representative intermediate firm

The representative intermediate firm \(i \in [0, 1]\) has the following technology,

\[
X_{i,t}(i) = e^{\epsilon^A_i} K_{ti,t}^a(i) H_{i,t}(i)^{1-\alpha}
\]

where \(X_{i,t}(i)\) is the production function of the intermediate good that combines capital \(K_{ti,t}^a(i)\), labour demand \(H_{i,t}(i)\) to household and technology \(e^{\epsilon^A_i}\). Here, \(e^{\epsilon^A_i}\) is an
AR(1) productivity shock. We assume perfect competition on the intermediate producer segment, so that it maximises its profit \((\Pi^H_{i,t}(i))\),

\[
\Pi^H_{i,t}(i) = P^x_{i,t} X_{i,t}(i) - Z_{i,t} K^u_{i,t}(i) - W_{i,t} H_{i,t}(i)
\]

subject to the production function. In this expression, \(P^x_{i,t}\) is the price of the intermediate good, \(Z_{i,t}\) is the remuneration of capital and \(W_{i,t}\) the nominal wage paid by the representative firm. As the marginal cost is the same across firms, the nominal price charged by the intermediate firm is,

\[
P^x_{i,t}(i) = P^x_{i,t} = \frac{1}{\epsilon^{i,t}} \left( \frac{Z_{i,t}}{\alpha} \right) \left( \frac{W_{i,t}}{1 - \alpha} \right)^{(1 - \alpha)} \tag{5}
\]

The representative entrepreneur

Each intermediate firm hires labour freely, but requires funds to finance the renting of capital needed to produce the intermediate good. We assume that each intermediate firm \(i \in [0, 1]\) is closely associated to a given entrepreneur \(e \in [0, 1]\), which aims at financing the capital renting of the intermediate firm. In real terms of the consumption basket, the amount of capital to be financed by the representative firm is equal to \(Q_{i,t} K_{i,t+1}(i)\), where \(Q_{i,t}\) is the real price of capital. This quantity is financed by two means: the net wealth of the entrepreneur \(e\), \(N_{i,t}(e)\), and the amount that is borrowed by the entrepreneur from the banking system, \(L^d_{i,t+1}(e)\). We also add lending demand habits \(h^e_{i;t}\) to fit the data implying that \(L^H_{i,t+1}(e) = \frac{L}{(L-h^e_{i;t})} (L^d_{i,t+1}(e) - h^e_{i;t} L^d_{i,t}(e))\). Thus, the entrepreneur balance sheet writes,

\[
Q_{i,t} K_{i,t+1}(i) = L^H_{i,t+1}(e) + P^x_{i,t} N_{i,t+1}(e) \tag{6}
\]

We assume that the project of the intermediate firm is risky. To modelize individual riskiness, we correct the expected aggregate return of project \(R^k_{i,t+1}\) with an individual random value \(\omega_{i,t+1}(e)\), drawn from a Pareto distribution; namely, for entrepreneur \(e \in [0, 1]\), the \text{ex post} gross return of its individual project is, \(\omega_{i,t+1}(e) R^k_{i,t+1}\). Since he must repay to the bank \(L^H_{i,t+1}(e)\), at the gross price \(P^L_{i,t}\), the \text{ex post} net return of the project is \(\omega_{i,t+1}(e) R^k_{i,t+1} Q_{i,t} K_{i,t+1}(e) - P^L_{i,t} L^H_{i,t+1}\). When the entrepreneur undertakes his decision, he ignores the value of \(\omega_{i,t+1}(e)\). We assume that the level of the individual profitability affects the survival of the entrepreneur: for a high realisation of \(\omega_{i,t}(e)\) (namely \(\omega_{i,t}(e) > \omega^C_{i,t}\), where \(\omega^C_{i,t}\) is a critical level of profitability that is endogenously determined below in the economy) the entrepreneur is able to repay its loan and survives; for a low realisation of \(\omega_{i,t}(e)\) (namely \(\omega_{i,t}(e) < \omega^C_{i,t}\)) the entrepreneur goes bankrupt and does not make any repayment to the banking system. As will be shown below, the critical point \(\omega^C_{i,t}(e)\) is determined by the macroeconomic features of the model. Thus, the expected profitability of entrepreneur \(e\) is,

\[
E_t \{ \Pi^E_{i,t+1}(e) \} = \eta^s_{i,t+1} \omega_{i,t+1}(e) R^k_{i,t+1} Q_{i,t} K_{i,t+1}(e) - P^L_{i,t+1} L^H_{i,t+1}(e)\]

where \(\eta^s_{i,t+1}\) is the survival probability of the entrepreneur. Here \(\eta^s_{i,t+1} \omega_{i,t+1}(e)\) is the conditional expectation of \(\omega_{i,t+1}(e)\) assuming that the entrepreneur has enough profitability to repay its loan. Pareto distribution computation are given in appendix B.1.

We assume that the entrepreneur is risk adverse. Namely, he tends to underestimate the individual profitability of its project. To take into account this particular feature, we introduce a given function \(g(\omega)\) to modelize the entrepreneur risk aversion. This
function is such that, \( \forall \omega \in [0; 1], g(\omega) < \omega \). We assume that \( g(\omega) = \tilde{\gamma}^E \omega^{\frac{\omega}{\gamma E + 1}} \), so that the isoelastic function \( g(.) \) for positive values of parameters \( \tilde{\gamma}^E \) and \( \omega \), the agent is risk averse\(^2\). We add an exogenous shock in the aversion function to account for changes in the expected profitability of financial projects \( g \left( \omega_{i,t+1} (e) \right) e^{\xi_{i,t+1} / \gamma} \). Thus, the expected gains that takes into account \( g(.) \) is less than the standard expected gain, which means that the entrepreneur is pessimistic regarding the rentability of its investment. Thus, the entrepreneur chooses a capital value of \( K_{i,t+1} (e) \) that maximises its expected profit,

\[
\max_{\{K_{i,t+1}(e)\}} E_t \left\{ \eta_{i,t+1}^g \left\{ E_t \left\{ \left( \omega_{i,t+1} (e) \right) e^{\xi_{i,t+1} / \gamma} \right\} R_{i,t+1}^d Q_{i,t} K_{i,t+1} (e) - P_{i,t}^L L_{i,t+1}^d (e) \right\} \right\}.
\]

The first order solution of the entrepreneur optimizing program defines the expected external finance premium \( (E_t \{ P_{i,t+1}^k / P_{i,t}^L \}) \) as,

\[
E_t R_{i,t+1}^k \frac{1}{P_{i,t}^L} = \frac{\tilde{\gamma}^E}{E_t \omega_{i,t+1} (e) e^{\xi_{i,t+1} / \gamma} \left( \frac{\omega_{i,t+1} (e) e^{\xi_{i,t+1} / \gamma}}{\gamma} \right)^{\frac{\gamma}{\gamma + 1}}}. \tag{7}
\]

This premium captures the extra remuneration needed by the entrepreneur to undertake the decision to finance the investment of the intermediate firm. The interest rate spread and the accelerator phenomenon disappear if \( \omega = 0 \). We assume that the entrepreneur cannot make an arbitrage with a riskless asset. Thus the net wealth of the entrepreneur in the next period is equal to,

\[
N_{i,t+1} (e) = \frac{\tau^E}{e^{\xi_{i,t}}} \Pi_{i,t}^E (e)
\]

where \( e_{i,t}^N \) is an exogenous process of net wealth destruction. Once the total amount of borrowing is decided, the entrepreneur chooses to borrow either at home banks or abroad. The total amount of loans undertaken by the representative entrepreneur writes,

\[
L_{i,t+1}^d (e) = \left( \left( 1 - \alpha_i^L \right)^{1/\nu} L_{h_i,t+1}^d (e)^{\left( \nu - 1 \right)/\nu} + \left( \alpha_i^L \right)^{1/\nu} L_{f_i,t+1}^d (e)^{\left( \nu - 1 \right)/\nu} \right)^{\nu/(\nu - 1)},
\]

with parameter \( \nu \) representing the elasticity of substitution between domestic and foreign loans, \( \alpha_i^L \) the degree of bank integration in the monetary union and \( L_{f_i,t+1}^d (e) \) the amount of cross border loans demanded by entrepreneur \( e \) in country \( i \). The total cost of loans is thus defined according to,

\[
P_{i,t+1}^L = \left( \left( 1 - \alpha_i^L \right) \left( R_{h_i,t+1}^L \right)^{1-\nu} + \alpha_i^L \left( R_{f_i,t+1}^L \right)^{1-\nu} \right)^{1/(1-\nu)},
\]

and the relative loan demands are thus defined according to,

\[
L_{h_i,t+1}^d (e) = \left( \left( 1 - \alpha_i^L \right) \left( R_{h_i,t+1}^L \right)^{-\nu} L_{i,t+1}^d (e) \right)^{-\nu} \text{ and } L_{f_i,t+1}^d (e) = \alpha_i^L \left[ \frac{R_{f_i,t+1}^L}{P_{i,t+1}^L} \right]^{-\nu} L_{i,t+1}^d (e).
\]

\(^2\) \( \tilde{\gamma}^E = \omega^1 - \frac{\omega}{\gamma + 1} \) is a rescale parameter in order to have a steady state independant of \( \omega \).
The representative capital supplier

The third agent of the intermediate production sector are the suppliers of capitals that lend capital to the intermediate firms, once it is financed by the entrepreneurs. Capital suppliers are homogeneous and distributed over a continuum normalised to one. The representative capital supplier \( k \in [0; 1] \) acts competitively to supply a quantity of capital \( K_{i,t+1} (k) \) to intermediate firms and invest a quantity of final goods \( I_{i,t} (k) \) to keep it productive. We assume that it is also costly to invest (i.e. it has to pay an adjustment cost on investment \( AC_{i,t} (k) = \chi_{i,t} f \left( e^{\delta_{i,t}} I_{i,t} (k), I_{i,t-1} (k) \right) \) where \( \chi_{i,t} \) is a parameter and \( f \) is the investment adjustment cost function. The functional form of \( f \) is presented in appendix. Thus the capital stock of the representative capital supplier evolves according to, \( K_{i,t+1} (k) = (1 - AC_{i,t} (k)) I_{i,t} (k) + (1 - \delta) K_{i,t} (k) \). The capital producer produces the new capital stock \( Q_{i,t} K_{i,t+1} (k) \) by buying the depreciated capital and investment goods. The project of the representative supplier thus writes,

\[
\Pi_{i,t}^k (k) = Q_{i,t} K_{i,t+1} (k) - (1 - \delta) Q_{i,t} K_{i,t} (k) - P_{i,t}^I I_{i,t} (k),
\]

where \( I_{i,t} (k) \) is,

\[
I_{i,t} (k) = \left( (1 - \alpha_{i,t})^{1/\mu} I_{hi,t} (k)^{\mu-1/\mu} + (\alpha_{i,t})^{1/\mu} I_{fi,t} (k)^{\mu-1/\mu} \right)^{\mu/(\mu-1)}.
\]

In this expression, parameter \( \mu \) is the elasticity of substitution between domestic and foreign goods in investment and \( \alpha_{i,t} \) measures the degree of investment diversification in the monetary union between home and foreign countries. We assume a national bias in investment choices so that, \( \alpha_{i,t} < 0.5 \). The price index of investment is,

\[
P_{i,t}^I = \left( (1 - \alpha_{i,t}) (P_{hi,t})^{1-\mu} + \alpha_{i,t} (P_{fi,t})^{1-\mu} \right)^{1/(1-\mu)}.
\]

The representative capital supplier chooses \( I_{i,t} (k) \) to maximize profits,

\[
\max_{\{ I_{i,t}(k) \}} E_t \left\{ \sum_{t=0}^{\infty} \beta^t \frac{\lambda_{i,t+1}}{\lambda_{i,t}} \Pi_{i,t}^k (k) \right\},
\]

where \( \beta^t \lambda_{i,t+1} / \lambda_{i,t} \) is the household subjective discount factor. The price of capital renting thus solves,

\[
Q_{i,t} = P_{i,t}^I + Q_{i,t} \frac{\partial (I_{i,t} AC_{i,t}^I)}{\partial I_{i,t}} + \beta \frac{\lambda_{i,t+1}}{\lambda_{i,t}} Q_{i,t+1} \frac{\partial (I_{i,t+1} AC_{i,t+1}^I)}{\partial I_{i,t}}.
\]

Ignoring investment adjustment costs in this last expression (i.e. imposing \( \chi^I = 0 \)), we simply get, \( Q_{i,t} = P_{i,t}^I \). \( Q_{i,t} \) stands for the asset price given the adjustment costs on investment production function. As in Smets and Wouters, we assume that capital requires one period to be settled so that, \( K_{i,t+1}^u = u_{i,t} K_{i,t} \) given a level of capital utilization of capital \( u_{i,t} \). Thus, the rentability from holding one unit of capital from \( t \) to \( t+1 \) is determined by,

\[
E_t R_{i,t+1}^k = \left( 1 + \chi^B \right) \left( B_{i,t+1} (j) - B_i (j) \right) \left[ E_t Z_{i,t+1} - P_{i,t} \Phi (u_{i,t+1}) + (1 - \delta) E_t Q_{i,t+1} \right] / Q_{i,t}
\]
where $\Phi(u_{i,t+1})$ is the capital utilization cost function. Following BGG, we lag the equation 9 to get the \textit{ex post return} of capital. As Smets Wouters, the optimal capital utilization determines the relationship between capital utilization and the marginal production of capital, $Z_{i,t+1}^{1-\psi_i} = u_{i,t+1}$, where $\psi_i \in [0; 1]$ is the elasticity of utilization costs with respect to capital inputs.

### 2.3 The banking sector

The banking sector acts monopolistically to provide loans to entrepreneurs. The total number of homogenous banks is normalised to one. The representative bank $b \in [0; 1]$ in country $i$ provides a quantity of loans $L_{i,t+1}^s(b)$ that is financed by loans from the central bank (with a one period maturity) at the interbank interest rate $R_t$.

The representative bank sets the rate of interest that has to be charged to the entrepreneur loan. We assume that banks ignore the individual \textit{ex ante} viability of borrowers. However we assume that banks know the distribution of individual projects in terms of $\omega(e)$ so that they can compute the expected value of earnings of the next period, depending on the state of nature (ie, whether the entrepreneur reimburses or does not reimburse). Thus, the expected profit is defined as,

$$ E_t \Pi_{i,t+1}^B(b) = E_t \bar{\eta}_{t+1}^s L_{i,t+1}^s(b) - \Omega_{i,t}^B(b) R_t L_{i,t+1}^s(b) $$

In this setting we assume that there is no discrimination between borrowers, so that the representative bank serves both domestic and foreign entrepreneur without taking into account specificities regarding the national viability of projects. Bank default expectation regarding entrepreneurs’ projects is defined as, $\bar{\eta}_{t+1}^s = (1 - \xi) \bar{\eta}_{H,t+1} + \xi \bar{\eta}_{F,t+1}$.

Moreover, we allow partial spread indexation $\Omega_{i,t}^B(b) = (s_{i,t-1}^B(b) / s^B(b))^{\xi^B}$ where $s_{i,t}^B(b) = R_{i,t}^L(b) / R_t$ is the banking spread (with steady state $s^B(b)$) and the level of partial indexation $0 \leq \xi^B \leq 1$. This indexation catches some imperfect interest rate pass-through. Each bank $b$ maximises profits under the demand constraint,

$$ L_{i,t+1}^s(b) = \left( \frac{R_{i,t}^L(b)}{R_{i,t}^L} \right)^{-\epsilon_b} L_{i,t+1}^s $$

the first order condition is,

$$ s_{i,t}^B(b) = \frac{R_{i,t}^L(b)}{R_t} = \left( \frac{\epsilon_b}{\epsilon_b - 1} \right) \frac{1}{E_t \bar{\eta}_{t+1}^s} \left( \frac{R_{i,t-1}^L(b)}{R_{t-1}} \right)^{\xi^B} s^B(b)^{-\xi^B} $$

so that each bank decides the size of the spread depending on the expected failure rate of its customers and the previous spread level. If $\xi^B = 0$, the model is very close to the BGG’s financial accelerator.

### 2.4 The final goods sector

Final producers are distributed over a continuum normalised to one. The representative final producer $z \in [0; 1]$ acts as a consumption bundler. It combines national intermediate goods to produce the final good $z$. The production function of the representative
firm is,
\[ Y_{i,t}(z) = \left( \int_0^1 X_{i,t} (i, z)^{1\over\alpha} \, di \right)^{\alpha \over 1 - \alpha}, \]
with input demand
\[ X_{i,t} (i, z) = \left( \frac{P_{x,i,t}(i)}{P_{i,t}(z)} \right)^{1 - \epsilon} Y_{i,t}(z). \]

Here, \( P_{x,i,t}(i) \) is the nominal marginal cost of intermediate firm and \( P_{i,t}(z) \) stands for the nominal price of final good \( Y_{i,t}(z) \). Assuming that the final firm is able to modify its selling price with a probability \( 1 - \theta_i \), the representative firm chooses \( \{ P^*_i(t) \} \) to maximize its expected sum of profits,
\[
\max_{\{ P^*_i(t) \}} \mathbb{E}_t \left\{ \sum_{\tau=0}^{\infty} (\theta_i \beta)^{\tau} \lambda_{i,t+\tau} \left[ (1 - \tau^y) P^*_i(t) \Xi_{i,t,t+\tau} - P_{x,i,t+\tau} \right] Y_{i,t+\tau}(z) \right\},
\]
under the demand constraint,
\[ Y_{i,t+\tau}(z) = \left( \frac{P^*_i(t)(z)}{\Xi_{i,t,t+\tau}} \right)^{1 - \epsilon} Y_{i,t+\tau}, \quad \tau > 0,
\]
where \( Y_{i,t} \) represents the quantity of the goods produced in country \( i \), \( \tau^y \) is a proportional tax income on final goods producers’ profits, \( \lambda_{i,t} \) is the household marginal utility of consumption and the indexation price is,
\[
\Xi_{i,t,t+\tau} = \left\{ \begin{array}{ll}
\prod_{j=1}^{\tau-1} (\pi_{i,t+\tau})^{\xi_i}, & j > 1 \\
1, & j = 0
\end{array} \right.
\]
where \( \Xi_{i,t,t+\tau} \) describes the fact that if the firm \( z \) does not reoptimize its price, it updates according the rule \( P_{i,t}(z) = \pi_{i,t+1} P_{i,t-1}(z) \). In order to fit better the data, we allow inflation persistence so that the fraction \( \theta_i \) of final firms who did not receive price signal change will partially index their nominal price to lagged inflation rate, where \( \pi_{i,t+1} \) is the inflation of aggregate final prices and \( \xi_i \in [0; 1] \) stands for the level of indexation.

The first order condition that defines the price of the representative final firm is,
\[ P^*_i(t)(z) = \frac{\mathbb{E}_t \left\{ \sum_{\tau=0}^{\infty} (\theta_i \beta)^{\tau} \lambda_{i,t+\tau} \left( \frac{Z_{i,t+\tau}}{\lambda_{i,t}} \right)^{\alpha} \left( \frac{W_{i,t+\tau}}{1 - \alpha} \right)^{(1-\alpha)} Y_{i,t+\tau}(z) \right\}}{(\epsilon - 1)(1 - \tau^y)} \frac{\mathbb{E}_t \left\{ \sum_{\tau=0}^{\infty} (\theta_i \beta)^{\tau} \lambda_{i,t+\tau} \Xi_{i,t,t+\tau} Y_{i,t+\tau}(z) \right\}}{\epsilon^{\xi_{i,t+\tau}}}.
\]

### 2.5 Authorities

National governments finance public spendings by charging a proportional tax \( \tau^y \) on final producer profits \( \int_0^1 \Pi_{i,t}^y(k) \, dk \) and by receiving a total value of taxes \( \int_0^1 T_{i,t}(j) \, dj \) from households. The buget constraint of the national government writes,
\[
\int_0^1 T_{i,t}(j) \, dj + \tau^y \int_0^1 \Pi_{i,t}^y(z) \, dz = G_{i,t} = \left( \int_0^1 G_{i,t}(z)^{1\over\alpha} \, dz \right)^{\alpha \over 1 - \alpha},
\]
where, $G_{i,t}$ is the total amount of public spending in the $i^{th}$ economy. The general expression of the interest rate implemented by the monetary union central bank writes,

$$R_t = R_{t-1}^B \left( \frac{P_{h,t}^c}{P_{h,t-1}^c} \right) \left( \frac{Y_{h,t}Y_{f,t}}{Y_{h,t-1}Y_{f,t-1}} \right) e^{\Delta y} e^{(1-\rho^y)}$$

where $\epsilon_t^R$ is an AR(1) monetary policy shock, $\phi^y$ is the inflation target parameter, $\phi^\Delta y$ is the GDP growth target, $\phi^E$ and $\phi^B$ estimates the response size of the monetary authority to financial distress as measured by movements in credit spreads (Çuardia & Woodford, 2010). In what follows we will assume alternative policy rule as special cases of this general expression: a standard Taylor rule (by imposing $\phi^E = \phi^B = \phi^L = 0$).

### 2.6 Aggregation and general equilibrium

We first have to determine the number of profitable projects financed by the banking sector. To separate the total number of contracts, there exists a critical value (a cut off point) defined as $\omega^C_{i,t} = \omega_{i,t}(e^c)$, where $e^c$ is the "critical" entrepreneur, such that the project just breaks even, i.e., $\omega_{i,t}(e^c)R_{i,t}^kQ_{i,t-1}K_{i,t}^1(e^c) = P_{i,t}^L L_{i,t}^H(e^c)$. Assuming $ex$ ante symmetry in the behaviour of entrepreneurs ($K_{i,t} = K_t$, $L_{i,t} = L_{t}^H$), we get the cut off point that discriminate profitable and non profitable projects,

$$\omega^C_{i,t} = \frac{P_{i,t}^L L_{i,t}^H}{R_{i,t}^kQ_{i,t-1}K_{i,t}}.$$

Thus using a pareto distribution ($\omega \sim \mathcal{P}(k;\omega_{\text{min}})$), we find a negative relation between $\omega$ and the default risk implying that a healthy firm has a lower probability to go bankrupt than a firm that is closer to $\omega_{\text{min}}$. The interest of using the Pareto distribution is that it can be presented in log deviation, so that we can get a simple expression for the dynamics of $\omega_{i,t}$, endogenously determined in the model (see B.1). We can then combine this value with the external finance premium (7) to get the link between it and the financial accelerator,

$$E_{i,t} = \left( \frac{P_{i,t}^L}{P_{i,t}^c} \right) \left( \frac{N_{i,t} + 1}{Q_{i,t}K_{i,t-1}^1} \right) e^{(1-\rho^y)}$$

The external finance premium is a positive function of the leverage ratio, $\frac{Q_{i,t}K_{i,t+1}}{N_{i,t+1}}$. Thus an increase in net wealth induces a reduction of the external finance premium. A shock that affects the entrepreneur net wealth will thus affects the rentability of the physical capital in the economy. The size of the accelerator phenomenon is determined by the degree of risk aversion: a higher value of risk aversion will imply a higher reaction of the external finance premium following a counter cyclical movement in net wealth. As the rentability of capital is a cost for the intermediate sector, a variation in the net wealth will have aggregate consequences on goods supply through the channel of the capital market. The amount of capital of defaulting entrepreneurs is consumed in terms of final goods $(1-\eta^{\delta}_{i,t})\omega_{i,t}R_{i,t}^kQ_{i,t-1}K_{i,t}^3$.

---

3We define $(1-\eta^{\delta}_{i,t})\omega_{i,t}$ as a $G(.)$ function à la BGG where the density function of $\omega_{i,t}$ is below the cut off value $\omega^C_{i,t}$: $G(\omega_{i,t}) = \int_{\omega_{\text{min}}}^{\omega^C_{i,t}} \omega_{i,t} f(\omega_{i,t}) d\omega_{i,t}$. 

---
The aggregation of prices of the final goods sector leads to the expression of the Phillips curve. Following the literature (Gal& Gertler, 1999), we take into account a smoothing parameter in the Phillips curve. We assume that the firms that cannot reoptimize their selling price will partly index them on the past inflation rate by following the expression, $P^*_{i,t}(z) = \left(\frac{P_{i,t-1}}{P_{i,t-2}}\right)^{\xi_i} P^*_{i,t-1}(z)$, with parameter $\xi_i$ representing the indexation level of prices. Thus, the national price of the national goods (ie, the GDP deflator) is defined as,

$$(P_{i,t})^{1-\epsilon} = \theta_i \left[ P_{i,t-1} \left( \frac{P_{i,t-1}}{P_{i,t-2}} \right)^{\xi_i} \right]^{1-\epsilon} + (1 - \theta_i) \left( P^*_{i,t} \right)^{1-\epsilon}$$

We define the aggregate values of labour supply, $H_{i,t} = \int_0^1 H_{i,t}(j) \, dj$, consumption, $C_{i,t} = \int_0^1 C_{i,t}(j) \, dj$, capital supply, $K_{i,t} = \int_0^1 K_{i,t}(k) \, dk$, loan supply, $L^*_t = \int_0^1 L^*_t(b) \, db$,

intermediate production, $X_{i,t} = \int_0^1 X_{i,t}(i) \, di$, bonds holdings $B_{i,t} = \int_0^1 B_{i,t}(j) \, dj$,

final production, $Y_{i,t} = \int_0^1 Y_{i,t}(z) \, dz$, investment, $I_{i,t} = \int_0^1 I_{i,t}(k) \, dk$. We define the rate of production inflation, and of consumer price inflation, $(\pi_{i,t}, \pi_{f,t}, \pi^*_h, \pi^*_f)$, $\pi_{i,t} = P_{i,t}/P_{i,t-1}$, the terms of trade $TT_t = P_{f,t} - P_{h,t}$.

Thus, in this setting, given the realization of 13 shocks $\{S_t\}_{t=0}^\infty = \{\varepsilon_{i,t}, \varepsilon_{i,t}^A, \varepsilon_{i,t}^G, \varepsilon_{i,t}^Q, \varepsilon_{i,t}^N, \varepsilon_{i,t}^R\}_{t=0}^\infty$ (recalling that $i \in \{h, f\}$), a competitive equilibrium is defined as a sequence of quantities,

$$\{Q_t\}_{t=0}^\infty = \{Y_{h,t}, Y_{f,t}, C_{h,t}, C_{f,t}, X_{h,t}, X_{f,t}, H_{h,t}, H_{f,t}, K_{h,t}, K_{f,t}, I_{h,t}, I_{f,t}, L^*_h, L^*_f, B_{h,t}, B_{f,t}, N_{h,t}, N_{f,t}, u_{h,t}, u_{f,t}\}_{t=0}^\infty,$$

and a sequence of prices,

$$\{P_t\}_{t=0}^\infty = \{P^*_{h,t}, P^*_{f,t}, P_{h,t}, P_{f,t}, P^*_{h,t}, P^*_{f,t}, W_{h,t}, W_{f,t}, R_{h,t}, R_{f,t}, R^*_h, R^*_f, R^*_L, Q_{h,t}, Q_{f,t}, Z_{h,t}, Z_{f,t}, P^L_{h,t}, P^L_{f,t}, R_t, \pi_{h,t}, \pi_{f,t}, \pi^*_h, \pi^*_f\}_{t=0}^\infty,$$

such that for a given sequence of prices $\{P_t\}_{t=0}^\infty$, the realization of shocks $\{S_t\}_{t=0}^\infty$, the sequence $\{Q_t\}_{t=0}^\infty$ respects first order conditions for households and maximizes firms, bank... profits and for a given sequence of quantities $\{Q_t\}_{t=0}^\infty$, the realization of shocks $\{S_t\}_{t=0}^\infty$, the sequence $\{P_t\}_{t=0}^\infty$, guarantees labour and capital market equilibriums,

$$\int_0^1 H_{i,t}(j) \, dj = \int_0^1 H_{i,t}(i) \, di \text{ and } \int_0^1 K_{i,t}(i) \, di = \int_0^1 K_{i,t}(k) \, dk$$

loan market equilibrium,

$$L^*_h + L^*_f = \int_0^1 L_{h,t}(e) \, de + \int_0^1 L_{f,t}(e) \, de$$

intermediate goods market equilibrium,

$$X_{h,t} = \left( \frac{P^*_{h,t}}{P_{h,t}} \right)^{-\epsilon} \left( \frac{P_{h,t}}{P^c_{h,t}} \right)^{-\mu} Y_{h,t} \text{ and } X_{f,t} = \left( \frac{P^*_{f,t}}{P_{f,t}} \right)^{-\epsilon} \left( \frac{P_{f,t}}{P^c_{f,t}} \right)^{-\mu} Y_{f,t},$$
final goods market equilibrium,

\[
Y_{h,t} = (1 - \alpha^C) \left( \frac{P_{h,t}}{P_{h,t}} \right)^{-\mu} C_{h,t} + \alpha^C \left( \frac{P_{h,t}}{P_{f,t}} \right)^{-\mu} C_{f,t} \\
+ (1 - \alpha^I) \left( \frac{P_{h,t}}{P_{h,t}} \right)^{-\mu} \left( 1 - AC_{h,t}^I \right) I_{h,t} + \alpha^I \left( \frac{P_{h,t}}{P_{f,t}} \right)^{-\mu} \left( 1 - AC_{f,t}^I \right) I_{f,t} \\
+ G_{h,t} + AC_{h,t}^B + \left( 1 - \eta_{h,t}^s \right) \omega_{h,t} Q_{h,t} K_{h,t}^\nu + \Phi(u_{i,t}) K_{h,t-1}, \omega \\
\]

\[
Y_{f,t} = \alpha^C \left( \frac{P_{f,t}}{P_{h,t}} \right)^{-\mu} C_{h,t} + (1 - \alpha^C) \left( \frac{P_{f,t}}{P_{f,t}} \right)^{-\mu} C_{f,t} \\
+ \alpha^I \left( \frac{P_{f,t}}{P_{h,t}} \right)^{-\mu} \left( 1 - AC_{h,t}^I \right) I_{h,t} + \alpha^I \left( \frac{P_{f,t}}{P_{f,t}} \right)^{-\mu} \left( 1 - AC_{f,t}^I \right) I_{f,t} \\
+ G_{f,t} + AC_{f,t}^B + \left( 1 - \eta_{f,t}^s \right) \omega_{f,t} Q_{f,t} K_{f,t}^\nu + \Phi(u_{f,t}) K_{f,t-1}, \\
\]

and the international financial market equilibrium,

\[ CA_{h,t} + CA_{f,t} = 0, \]

where,

\[
CA_{h,t} = (B_{t-1} - B_t) + [(L_{h,f,t+1} - L_{h,f,t}) - (L_{f,h,t+1} - L_{f,h,t})] \\
= P_{h,t} (Y_{h,t} - G_{h,t} - AC_{h,t}) - (P_{c,h,t} C_{h,t} + P_{h,t}^I I_{h,t}) \\
+ (R_{t}^B - 1) B_t + [(R_{h,t}^L - 1) L_{h,f,t} - (R_{f,t}^L - 1) L_{f,h,t}] \\
\]

3 Estimation

In this section, we discuss the data, the calibrated parameters and the priors and then we give the parameters estimates. The model is estimated with Bayesian methods. We focus our empirical analysis on the two major countries of the EMU, Germany and France, that are both home and host of cross border banking flows\(^4\). To keep the model as simple as possible, we suppose a symmetric steady states between those countries in order to get a long term balanced current account. This way, we estimate parameters that drive the model dynamics while we calibrate those determining the steady state.

3.1 Data

The data is quarterly and spans the period from 2003Q1 to 2012Q3, it includes 13 times series for Germany and France: namely, real GDP, real consumption, real investment, the ECB refinancing operation rate, the GDP deflator, the outstanding amount of loan and lending rate to non financial corporations. Since data for wages and hours worked

\(^4\text{France and Germany represents 49\% of the euro area's GDP on average between 1999 and 2012 and 45\% of the population.}\)
are not available for euro area members, the model does not include the calvo wage setting. See appendix A for a description of the dataset. Data with a trend are made stationary using a linear trend and are divided by the population. We also demean the data because we do not use the information contained in the observables mean. Figure 2 plots the transformed data.

![Figure 2: Observable variables used in the estimation](image)

### 3.2 Calibration and Prior Distribution of Parameters

#### Calibrated Parameters

We fix a small number of parameters commonly used in the literature of real business cycles models in table 1, these include the quarterly depreciation rate $\delta$, the quarterly discount factor $\beta$, the capital share in the production $\alpha$, the steady state of government expenditures in output $G/Y$ and the adjustment cost on portfolio $X_B$ (Schmitt-Grohé & Uribe, 2003). We also approximate the degrees of openness $\alpha^C$ and $\alpha^I$ using average intra–zone openness (Eyquem & Poutineau, 2010).

Regarding financial parameters, we fix the leverage ratio on the average loan-to-value for Germany (0.75) and France (0.7) (Gerali et al., 2010), while the spread between the lending rate and the refinancing rate is calculated on the average observable variables used in the estimation for France and Germany and has a value of 210 points basis annually, this is consistent with the literature (Faia, 2007). The annual entrepreneur failure rate of 1.8% is deducted from the lending spread, which is comparable to BGG.

At last the we suppose that in the long term banks act competitively, this assumption provides a very simple linear steady state along the Pareto distribution of $\omega \in [\omega_{\min}; +\infty]$. Recall that $\omega \sim P(\kappa; \omega_{\min})$ where $\kappa$ is the shape parameter and $\omega_{\min}$ the minimum possible value of $\omega$. When $\omega$ is at its lower bound ($\omega = \omega_{\min}$), the economy is riskless implying $R^k = R^l = R$ so that when $\omega > \omega_{\min}$ there are financial frictions and defaulting entrepreneurs in the steady state. Given the first order condition of banks and the survival rate of entrepreneurs written $\eta^* = (\omega_{\min}/\omega)^\kappa$, we can compute $\kappa$ and $\omega_{\min}$ via the following condition $\omega_{\min} = (\kappa - 1)/\kappa = 1 - N/K$. Calibrating the model without financial frictions ($\omega = \omega_{\min}$) and without loans ($L = 0$) makes the model really close to the Smets and Wouters model.
Table 1: Calibration of the model (all parameters are quarterly)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>β</td>
<td>0.99</td>
<td>Discount Factor</td>
</tr>
<tr>
<td>δ</td>
<td>0.025</td>
<td>Depreciation rate</td>
</tr>
<tr>
<td>α</td>
<td>0.36</td>
<td>Capital share</td>
</tr>
<tr>
<td>( \lambda^B )</td>
<td>0.07%</td>
<td>Portfolio adjustment costs</td>
</tr>
<tr>
<td>( G/Y )</td>
<td>0.2</td>
<td>Ratio government expenditures to gdp</td>
</tr>
<tr>
<td>( K/N )</td>
<td>1/0.275</td>
<td>Leverage ratio</td>
</tr>
<tr>
<td>( R^L - R )</td>
<td>0.0210 ( 0.25 )</td>
<td>Loan spread</td>
</tr>
<tr>
<td>( 1 - \alpha^C )</td>
<td>1 - 0.0922</td>
<td>Consumption home bias</td>
</tr>
<tr>
<td>( 1 - \alpha^I )</td>
<td>1 - 0.0439</td>
<td>Investment home bias</td>
</tr>
</tbody>
</table>

Prior Distributions

Our priors are listed in table 2. They are consistent with the bayesian estimation of DSGE models. For a majority of new keynesian model parameters, i.e. \( \sigma^\varepsilon_i \), \( \sigma^I_i \), \( h^\varepsilon_i \), \( \xi_i \), \( \lambda^I_i \), \( \psi_i \), \( \phi^\varepsilon \), \( \phi^\Delta y \) and shocks processes parameters, we use the prior distributions chosen by Smets Wouters (Smets & Wouters, 2007, 2003). Concerning international macroeconomic parameters, our priors are largely inspired by Lubik and Schorfheide (Lubik & Schorfheide, 2006), loan market openness \( \alpha^L \) has a beta prior of mean 0.12 and standard deviation of 0.05. Following these authors, we assume that the calvo parameter \( \theta_i \) is centered at 0.75 with a standard deviation of 0.05. We set the prior for the elasticity of the external finance premium \( \zeta_i \) to a beta distribution with prior mean equal to 0.05 and standard deviation 0.02 (Gilchrist et al., 2009).

Regarding the monetary policy rule, as the ECB mandate is to stabilize inflation in the euro area, the parameters related to the financial distress should be equalized to zero \( \phi^E = \phi^B = \phi^L = 0 \) in equation (10) to be near to the reality. However we want to simulate the response of the system to see how history would have evolved if ECB had financial stability mandate in its monetary policy rule. To do this exercise, we set opinionated priors in order to get positive and significant values for \( \phi^E \), \( \phi^B \) and \( \phi^L \). The spread parameters have a priors mean of 0.5 with standard error of 0.1 and credit growth parameter has the same prior than \( \phi^\Delta y \).

Methodology and 3 Different Scenarios

The methodology is standard to the bayesian estimations of DSGE models: the vectors of observables \( \mathcal{Y}_{t,obs} \) and measurement equations \( \mathcal{Y}_t \) are defined as (recalling that \( i \in \{h, f\} \))

\[
\mathcal{Y}_{t,obs} = [\Delta \log \hat{Y}_{t,t}, \Delta \log \hat{C}_{i,t}, \Delta \log \hat{I}_{t,t}, R^IB_t, \Delta \log D_e f_{t,GDP}^G, \Delta h^s_{t,t}, R^L_{t,t}]'
\]

\[
\mathcal{Y}_t = [\hat{y}_{i,t} - \hat{y}_{i,t-1}, \hat{c}_{i,t} - \hat{c}_{i,t-1}, \hat{i}_{i,t} - \hat{i}_{i,t-1}, 4 \times \hat{r}^IB_t, \hat{\pi}_{i,t}, \hat{l}^s_{i,t} - \hat{l}^s_{i,t-1}, 4 \times \hat{l}^L_{i,t}]'
\]

where \( \Delta \) denotes the temporal difference operator, \( \hat{X}_t \) is per capite variable of \( X_t \).

\(^{5}\)Some authors simulate responses with \( \phi^E \) equalized to 0.5 and 1 (Cúrdia & Woodford, 2010; Hirakata et al., 2011a).
The model matches the data setting $y_{t}^{\text{obs}} = y_{t}^{\text{r}} + y_{t}^{f}$ where $y_{t}^{\text{r}}$ is the vector of the mean parameters, we suppose this is a vector of all 0. Interest rates data are associated with one-year maturity loans, we take into account this maturity by multiplying by 4 the rates in the measurement equation. The number of shocks and observables variables are the same to avoid stochastic singularity issue. The structural shock processes are given in log-linearized form by an univariate representation. There are 5 country specific structural shocks for $i \in \{h, f\}$ and a common interest rate shock, 

$$
\varepsilon_{i,t}^{s} = \rho_{i}^{s} \varepsilon_{i,t-1}^{s} + \varepsilon_{i,t}^{s}, \quad \forall s = \{\beta, A, G, I, Q, N\}, \text{ and } \epsilon_{t}^{R} = \rho^{R} \epsilon_{t-1}^{R} + \varepsilon_{t}^{R}
$$

where $\rho_{i}^{s}, \rho_{i}^{A}, \rho_{i}^{G}, \rho_{i}^{I}, \rho_{i}^{Q}, \rho_{i}^{N}$ and $\rho^{R}$ are autoregressive roots of the exogenous variables, $\varepsilon_{i,t}^{\beta}, \varepsilon_{i,t}^{A}, \varepsilon_{i,t}^{G}, \varepsilon_{i,t}^{I}, \varepsilon_{i,t}^{Q}, \varepsilon_{i,t}^{N}$ and $\varepsilon_{t}^{R}$ are standard errors that are mutually independant, serially uncorrelated and normally distributed with zero mean and variances $\sigma_{i,\beta}^{2}, \sigma_{i,A}^{2}, \sigma_{i,G}^{2}, \sigma_{i,I}^{2}, \sigma_{i,Q}^{2}, \sigma_{i,N}^{2}$ and $\sigma_{R}^{2}$ respectively.

The posterior distribution combines the likelihood function with prior information. To calculate the posterior distribution to evaluate the marginal likelihood of the model, the Metropolis-Hastings algorithm is employed. To do this, a sample of 350 000 draws was generated, neglecting the first half.

Considering $\Theta$ the vector of parameters of the model $M(\Theta)$ presented in section 2, we estimate $\theta$ a subset of $\Theta$ depending on three different models:

- $M_{1}(\theta)$ : the taylor rule is standard $\phi_{L}^{E} = \phi_{E} = \phi_{B} = 0$, and there is no cross border lending flows between countries $\alpha_{L} = 0$.
- $M_{2}(\theta)$ : the taylor rule is standard, but we introduce cross border lending flows between countries by estimating $\alpha_{L}$.
- $M_{3}(\theta)$ : we don’t set up restriction on $\theta$ and estimate $\phi_{L}, \phi_{E}, \phi_{B}$ and $\alpha_{L}$.

We expect that the log marginal likelihood of $M_{2}(\theta)$ is higher than $M_{1}(\theta)$ and $M_{3}(\theta)$ because $M_{2}(\theta)$ is the nearest model to the reality.

### 3.3 Posterior Estimates

The posterior parameters differences between Germany and France give rise to asymmetries or co-movements of business cycles as shown by table 2. Regarding consumption, the associated risk aversion and habits tends to be higher in France while intertemporal substitution is lower in France than Germany. Moreover the posterior mean of the average duration of the price contracts is roughly 8 months for both countries, but France exhibits a stronger price indexation. The investment adjustment costs and shock standard deviation are upper in Germany but France’s investment adjustment costs shocks are much persistent. Futhermore, it is more expensive for France to change its capital utilization than Germany. All these results are consistent with data since the empirical volatility of real per capitae production, consumption is higher in Germany unlike loans and investment which have a standard deviation lower in Germany than in France. The financial accelerator parameter $x_{i}$ is higher in France than in Germany, meaning that the amplification of financial shocks tends to be higher in France, this effect is coherent with figure 1b: the negative relationship between credit spread and investment growth is stronger in France. We will comment the models differences in the next sections.

17
4 Bayesian IRF

Once the model has been estimated, we evaluate its propagation mechanism under alternative scenarios. We proceed in two steps: first we evaluate the consequences of cross border lending in the transmission of shocks. In a second step we evaluate the benefits of an extended taylor rule in a situation of cross border lending. In what follows, we concentrate on three main shocks: (1) an asymmetric productivity shock affecting the domestic economy; (2) an asymmetric financial shock that reduces the net worth of domestic entrepreneurs; (3) an asymmetric financial shock that negatively impacts the domestic interest rate spread between the profitability of investment and the lending rate. In this section we study the dynamics of the linearized model using bayesian impulse response function.

Regarding financial shocks, the decrease in the entrepreneur net worth can be considered as a shortcut to modelize the consequences of a stock market collapse that reduces the value of entrepreneur wealth. In contrast, a negative shock on the external finance premium can be related to problems associated with financial intermediaries (Friedman, 2013). The first term of the spread reflects the greater willingness to borrow when expected future income is higher, while the second term reflects the negative influence of the relevant real interest rate on optimal borrowing, all else equal. Thus, an external reduction of this spread can be associated with a deterioration of the borrowing conditions in the economy.

4.1 The consequence of Cross border banking

By comparing the two models $M_1$ and $M_2$, we study how cross border banking activities affect the propagation mechanism following asymmetric shocks. We consider Germany as the home country and France as foreign.

The consequences of an asymmetric positive total factor productivity shock

First, we study the transmission of a positive technology shock to assess the specificity of cross border lending with regard to more standard two country models that assume complete loan market segmentation. Figure 3 shows the simulated responses of the main macroeconomic and financial variables following a shock to $\epsilon_{i,t}$ equal in size to the standard deviation of total factor productivity estimated in table 2.

In the benchmark situation (represented with a dotted line) loan markets are segmented. As standardly documented in the literature, this productivity shock increases production, consumption and investment while decreasing the inflation rate in the domestic economy (Faia, 2007). The transmission of this shock to the foreign economy operates through both the current account and the reaction of the interbank interest rate of the central bank: the deterioration of the domestic terms of trade increases the relative competitiveness of domestic goods and the exports of domestic goods in the foreign economy. In the meanwhile, part of the increase in domestic total consumption falls on foreign goods, which tends to increase the price of foreign goods. As the decrease in domestic inflation is higher than the increase of the price of foreign goods, the average union wide rate of consumption price inflation decreases, which leads the central bank to reduce the interbank interest rate.
Regarding the financial adjustment of countries, we observe an increase in domestic leverage: as capital becomes more productive, firms invest more, which increases their demand for loans. Those results are quite standard with the financial accelerator literature (Bailliu et al., 2012). The interest rate on loans is driven by the leverage of entrepreneurs and the interbank rate. As leverage rises, this tends to increase the risk of firm bankruptcy and by so, the interest rate served by banks increases after 5 quarters, thus dampening the leverage effect in firms. Regarding loan creation in the domestic economy, there is a clear increase in loans distribution. Credit supply cycles are closely related to capital cycles, this explains why credit supply needs more than 30 periods to get back to its steady state.

The possibility of banks to engage in cross border loan distribution clearly affects the transmission of asymmetric productivity shocks in the monetary union. First regarding the macroeconomic variables, one should note that cross border bank lending acts as a mechanism that dampens the differential of activity between union members. Inflation is not affected with regard to the previous situation. Actually, the reaction of the domestic economy to the productivity shock is not affected with regard to the previous situation during the first two semesters. The main difference comes in the following quarters and could be associated to the contracting of foreign loans.

Regarding the novelty in international adjustment, one must note that the economy affected by the shock becomes a net importer of loans. The amount of loans contracted in this economy increases, while the total amount of loans distributed by the domestic banking system decreases. There is less loan creation in the domestic economy, as firms turn more easily to loan contracting with foreign banks with better borrowing conditions (as shown by the domestic loan index that is lower than previously at its minimum value that now becomes clearly negative). The increase in loan demand comes from the fact that the domestic loan rate index decreases more than previously, when the loan markets where segmented. The increase in loan demands increases the leverage of firms in the domestic economy. The dynamics of the current account become significantly different as it clearly deteriorates following the initial periods and becomes persistently in deficit. This adjustment should be linked to the fact that the domestic economy becomes a net importer of loans. This contracting of foreign loans induces a net increase in interest payments so that more resources are affected to the debt service of the economy (which explains that more resources are diverted from investment, as firms engage in loans contracting to finance the reimbursement of previous loan contracts).

With cross border lending, the consequences on the foreign economy are as follows: the possibility for the banking system to engage in cross border activities clearly impacts negatively the foreign economy. Indeed, as graphically reported the dynamics of foreign loan creation is now clearly affected by the exports of loans, and does not benefit to the foreign economy. Furthermore, the leverage effect is clearly negative. As foreign firms do not engage in loan contracting as much as it was the case under segmentation, the dynamics of foreign investment is clearly dampened, as shown in the last irf of figure 6. Thus foreign firms have access to less loans and investment in the foreign economy is relatively depressed. However as underlined by the figures reported in the graphs, the marginal negative impacts on activity, consumption and investment are higher in the domestic economy. By so, international differences in activity, consumption and investment are reduced. International bank flows thus do not increase the aggregate activity in the monetary union, they just promote more business cycle synchronization.
One must also note the increased persistence of shocks on the current account coming from cross border interest payments.

**A negative shock on firms net worth**

The second set of irfs reported in figure 4, describes the consequences of a negative shock on domestic firm net worth. This negative shock can be thought of as an overnight decrease in the value of investor capital (following, for example a stock exchange collapse).

**First, without cross border loans**, a reduction in the firms net worth depresses production and investment in the domestic economy, while it increases consumption. The inflation rate becomes negative. In this situation the reduction of activity is driven by the drop in investment. This dynamic of the real sector induces a negative inflation rate. The foreign economy economy also experiences a drop in inflation. The dynamics of inflation induces a reduction in the interbank interest rate, which in turn increases consumption and dampens the negative impact on domestic investment. As investment decreases more than activity, consumption increases and as the inflation rate in domestic product decreases more than the foreign economy, the terms of trade deteriorates and, as a consequence, more domestic goods are exported.

With segmented loan markets, the foreign economy is relatively preserved from the negative domestic financial shock. Indeed, it takes advantage of the interbank interest rate decrease. This reduces the rate of loan contracting, thus increasing loan demand, the leverage ratio and investment. Foreign consumption rise should be linked to the improvement in the foreign terms of trade which allows this economy to buy domestic goods cheaper.

**In this situation, cross border bank activity** acts as a dampening mechanism for the domestic economy but, at the union wide level, it acts as a transmission channel of the domestic financial crisis to the foreign economy. Indeed, now the foreign economy is affected by a clear reduction of loan creation that is driven by the clear increase of loan exportation to the domestic economy. One should notice that the reduction in the interest rate faced by domestic firms leads to an increase in domestic leverage, that implies less reduction in domestic investments, and, as consumption is not affected, leads to less reduction in activity and accordingly less impact on inflation. In the foreign economy less impact on inflation also, which leads the central bank to reduce less the interbank interest rate in the monetary union.

The transmission process acts as follows: with cross border bank lending, domestic firms have access to loans with a lower interest rate thus, they increase their loan demand to the foreign country. In the meanwhile, as there is a clear increase to the demand for foreign loans, the conditions of loan contracting in the foreign economy become more expensive for foreign firms. This, in turn, induces a drop in loan demand and depresses investment in the foreign economy. Here again, one should note that cross border bank lending dampens the difference in national activities, although, in this case, the phenomena is explained by the transmission of the domestic financial problems to the foreign economy.

Negative consequences of net worth shocks have already been studied in the literature with and without financial globalization (Hirakata et al., 2011b; Ueda, 2012). Our results are comparable to this literature.
A shock on the domestic external finance premium

We finally study the consequences of a negative shock on the domestic external finance premium, as presented in figure 5. We assume that the spread between the expected return on investment and the interest rate paid on loans becomes negative which means that, at the entrepreneur level, the interest rate paid on loans becomes higher than the prospective rate of return on capital investment (investment projects).

**When loan markets are segmented**, the negative external finance shock impacts negatively domestic investment and activity. As investment is more depressed than activity, consumption increases in the short run and inflation decreases to sustain consumption. This shock is transmitted abroad, as the relative price of domestic goods decreases. This leads to an increase in foreign consumption with a decrease in foreign activity and investment. To maintain the level of activity foreign inflation also decreases (however, by less than the domestic inflation). As the terms of trade of the domestic economy deteriorates, this leads to a current account surplus in the domestic economy. As inflation decreases at the union wide level, the central bank reduces its interbank interest rate, which in turn lowers both the domestic and foreign loan interest rate indexes. In the foreign economy the reduction in this index leads to a very slight increase in leverage, while it has no positive effect on domestic leverage. Indeed, this negative shock acts as a reduction in the prospective rentability of domestic firms, which lead them to reduce the use of loans in the financing of investment. As they have a negative appreciation of the rentability of investment project, they will not borrow even with a lower interest rate charged on loans. These system responses are consistent with the literature (Gilchrist et al., 2009).

**The possibility of cross border bank lending** does not affect the situation in the domestic economy, as it does not affect the preception of the rentability of investment projects: all macroeconomic variables are affected by the same dynamics than under the complete segmentation of loans markets. The possibility to make cross border loans affects the transmission towards the foreign economy. The main consequence can be observed in the graph depicting the adjustment of the foreign loan index. Now, as the domestic banking system can lend in the foreign country, foreign firms can borrow more with a cheaper interest rate. In this situation, the foreign leverage effect and investment increase. Accordingly, the domestic loan rate index decrease is damped, as a fraction of domestic loans are exported towards the foreign economy. This implies a more depressed evolution of domestic leverage. In the foreign economy, the amount of foreign loans decreases with regard to the segmented situation. Indeed, foreign firms borrow from domestic banks, which reduces the activity of foreign bank in the distribution of loans. The foreign economy clearly benefits from the depressed situation of domestic investment in borrowing at a very cheap interest rate loans from the domestic banking system.

4.2 The consequences of an extended taylor rule

In this section we assess the interest of an extended Taylor rule on the stabilisation of asymmetric real and financial shocks. By comparing the two models $M_2$ and $M_3$, we study how monetary policy interferes in the propagation mechanism following asymmetric shocks. We consider Germany as the home country and France as foreign. Under an
extended rule \((\mathcal{M}_3)\), the central bank responds negatively to an increase in the spread between the rentability of capital and the cost of borrowing. In this situation, by cutting the interbank rate it aims at reducing the interest rate on loans so promote borrowing at a lower interest rate, which is supposed to reduce the risks of investment. This makes the interbank interest rate reactive to the interest rate spread.

Taking the results obtained under the standard taylor rule as a benchmark \((\mathcal{M}_2)\), we evaluate the consequences of this extended rule on the IRFs \((\mathcal{M}_3)\). The posterior estimate of \(\mathcal{M}_3\) shows a higher standard deviation of shocks which constitutes a bias when comparing IRFs. To correct this standard deviation bias we do a counterfactual exercise like Smets and Wouters (Smets & Wouters, 2007) by distinguishing between the \textit{per se} consequences of the new interest rate rule (ie, by constraining the standard deviation of shocks to be equal to the value computed with the standard taylor rule) and the component affected by the evolution in the standard deviation of shocks under this new rule. We plot in figures 6, 7 and 7 the IRFs of the benchmark model \(\mathcal{M}_2\) with standard taylor rule (red dashed), the augmented taylor rule model \(\mathcal{M}_3\) with standard deviation bias (green dotted) and the counterfactual model with augmented taylor rule and fixed shocks (blue solid). We will compare the standard model with the counterfactual model.

As a main result, we find that the benefits arising from such a rule depends on the nature of shocks (in particular, this rule has not a \textit{per se} specific noticeable impact on macroeconomic variables). The main benefits are obtained if the economy is affected by financial shocks, as the extended rule provides an attenuation mechanism in the international transmission of this kind of shocks. This feature may be interesting since at the monetary union level, monetary policy can only target union wide variables. Introducing interest rate spreads may provide a simple solution to limit to a few countries the consequences of defavourable financial shocks, amplifying the business cycles of these countries, while preserving the economic health of the other members of the monetary union.

**Positive productivity shock**

As shown in figure 6 following an asymmetric productivity shock, the \textit{per se} effect of the extend rule is rather limited on the macroeconomic variables (see also (Bailliu et al., 2012)). The difference is mainly explained by the higher standard deviation of productivity shocks observed in the posterior distributions of parameters under an extended taylor rule. Aside from investment and activity in the domestic economy, the main differences are observed for the irfs of the financial variables: by correcting the spread the entrepreneur will borrow more from the bank, which increases the leverage ratio in the domestic economy inducing a slight transitory increase in both investment and activity with respect to the benchmark rule. As observed, there should be a reduction in the loan interest rate and an increase in loan demand associated to a higher reduction in interbank rate. However, in equilibirum, The increased demand for loans in the domestic economy (reflected in the domestic leverage ratio), leaves the the equilibirum loan interest rate index unaffected with respect to the benchmark situation. Regarding the demand side, noting is modified in the foreign economy. Regarding the supply of loans, both economies are affected by a net increase in loan creation that benefits to the domestic economy (as the foreign leverage irf remains unchanged with regard to the benchmark situation). Thus, the difference observed on the bayesian IRFs between the
benchark taylor rule and its extended version comes from the higher standard deviation of shocks computed by in the posterior estimates of the model parameters.

**Negative net worth shock**

As shown in figure 7 the impact of a reduction in the entrepreneurs’ net wealth is affected by the nature of the policy rule. Concerning the response for the domestic economy, similar results are available in the literature (Hirakata et al., 2011a). As found above, cross border lending operates as a destabilizing mechanism: this negative shock is transmitted negatively to the foreign country, that was initially protected with segmented loan markets. With cross border lending the \textit{per se} effect of the extended interest rate rule clearly dampens the negative impact on activity in the domestic economy while it positively impacts foreign activity. In the domestic economy the increase in consumption is higher, while investment remains unchanged with respect to the benchmark situation. The dynamics of consumption and activity lead to a positive inflation rate for domestic goods. The modification of the interest rate rule mainly impacts the situation of the foreign country, as now, we observe that its activity increases. Thus the foreign economy is preserved from the negative shock in the domestic economy. In the foreign economy, the loan rate index decreases with regard to the benchmark case, which in turn increases foreign leverage. The adoption of an extended rule mainly benefits to the foreign economy, by reducing the negative impact of the domestic shock on this economy. Regarding the supply of loans, the new rule mainly impacts the foreign economy as we observe less reduction in loan supply, that allows the foreign country to dampen the reduction in leverage. This rule computed on union wide averages clearly dampens the diffusion of the negative asymmetric financial shocks in a monetary union with cross border banking. Correcting this results by the increased value of the standard deviation reinforces this result: although it deteriorates the situation of the domestic economy (less activity, less consumption, a maintained investment through a higher leverage) and more inflation for domestic goods, it increases the stabilisation effect of the extended rule on the situation of the foreign economy (more activity, consumption and investment). Finally, As observed, this extended rule promotes a better synchronisation in the financial structure of firms (the negative impact on domestic leverage is not affected while the negative impact on foreign leverage is dampened).

**Positive spread shock**

The negative shock on the domestic external finance premium directly impacts the interbank interest rate in figure 8. Given the nature of the monetary policy rule the interest rate decreases, which clearly affects the macroeconomy through the consumption channel. As consumption increases while activity and investment are not specifically affected by the extended rule, production inflation clearly rises in the domestic economy. This rise in inflation is no longer corrected by monetary policy, as the central bank initially reacts to the interest rate spread by lowering its interest rate. The channel of investment is not impacted. Except for consumption, there is no specific noticeable impact on the domestic economy (either on the loan rate index or the leverage, because of the nature of the shock). As previously noted for the net worth shock, the foreign economy benefits from the extended rule. We can observe an increase in consumption and production, while investment is not significatively affected by the extension of
the interest rate rule. As previously observed, this extended rule promotes a better synchronisation in the financial structure of firms (reduction in the negative impact on domestic leverage and reduction in the positive increase of foreign leverage)

5 A quantitative analysis of the driving forces of business and credit cycles

In this section we perform a historical decomposition of the dynamics of the four main variable of interest (namely activity, investment, loan supply and the interest rate paid by borrowers), by contrasting the whole time period (2003Q1-2012Q3) and a sub sample corresponding to the post 2007 period. The aim of the exercise is to investigate how the financial crisis of 2007 has affected the driving forces of activity, investment and loan supplies. We first begin with a general evaluation of the contribution of the shocks over the whole sample period, while reducing the time period to 2007Q3-2012Q3 in a second step. As shown below, the financial crisis has induced an increased contribution of financial shocks in the explanation of the evolution of the main macroeconomic variables as well as an increased in the sensitivity of output fluctuations to the interbank interest rate. The structure of our model is also helpful to underline some major evolutions in the increasing role of foreign shocks as the driving forces of national variables after the financial crisis, as well as an increased heterogeneity in the determination of investment fluctuations between Germany and France.

5.1 The main driving forces of business and credit cycles over the sample period

Table 3 reports the posterior variance decomposition of activity, investment, loan supplies and of the average interest rate paid by borrowers in Germany and France over the whole sample period. As clearly shown, most of the variance of activity, investment and loan supplies are explained by real supply shocks. The role of financial shocks is only noticeable for the conditions of borrowing (ie, the interest rate paid by borrowers). Over the sample period, supply shocks explain more than 97% of the deviation of activity from its steady state value, more than 69% for investment and more than 78% for loan supplies in both economies. The contribution of real demand shocks either through public spendings (less than 1%), preferences or investment adjustment costs is negligible.

The contribution of financial shocks (either through the variance of net worth or the variance of the interest rate spread) is negligible for activity. The influence of financial factors on investment and loan supply is clearly different between Germany and France. Financial factors account for 14% of the fluctuation in german investment (while it only represents 2.73% in france) and for 6.30% of german loan supply (4.04% in france). Over the sample period, the main influence of financial factors should be found on the evolution of interest rate faced by borrowers. Indeed, between 2003Q1 and 2012Q3, financial factors account for 62.92% of the variance of german rate and 75.93% of french rates.

Two further points should be noticed in Table 3. First, the contribution of the interbank rate is only noticeable for the determination of the interest rate on loans
(around 5% of the variance of loan interest rates is explained by the interbank interest rate in the two economies). Second, national shocks have a very little impact on the other country variables. The only noticeable exception concerns the international diffusion of productivity shocks on the other country loan supply, which proves to be significant. Indeed, foreign productivity shocks appear as the second contributor (after the national productivity shock) to the variance of loan supply. Indeed, as documented in Table 3, french productivity shocks account for 13.23% of the variance of german loan supply while german shocks account for 7.45% of the variance of french loan supply. This result is in line with the increased cross border lending between EMU members such as Germany and France, since it shows that part of the variability of a country loan supply is explained by the need of the other country to finance the consequences of its own productivity gains.

These general findings can be completed with a graphical analysis of the contribution of the shocks on the evolution of activity, investment and loans supplies, on a quarter to quarter basis in figure 9. Before going into the details of the results, it should be noticed that the explanatory power of the model seems higher for Germany, as the residual contribution is much lower than for France (in particular regarding the deviation of investment and credit). The graphical representation of GDPs confirms the key role of real supply shocks as the main contributor to the deviation of this variable from its steady state value. This is observed both for the rise of GDPs up to 2007 and for the fluctuations observed after the financial crisis. Financial shocks affect strongly negatively this evolution after 2008. Furthermore, demand shocks seem to affect more the german GDP. The contribution of financial factors on investment fluctuations appears clearer on a quarter to quarter basis. Between 2003 and 2008, we observe a positive correlation between the evolution of german and french investment and both productivity and financial shocks. However the contribution of financial factors is still positive up to 2008 and the collapse of investment following the financial crisis is mainly explained during this period by the clear negative contribution of real supply shocks. The model catches up the financial crisis in the eurozone of 2008 via a strong negative contribution of financial factors on investment for both countries, this financial distress is also presented in figure 1b. Regarding the evolution of loans, France’s credit cycles are more affected by financial shocks on a quarter to quarter basis than Germany. Furthermore supply shocks and monetary policy play a strong role in explaining credit cycles of the eurozone while foreign and demand shocks are almost insignificant.

5.2 The consequences of the financial crisis: a sub-sample analysis

To evaluate how the financial crisis has impacted the two economies, we now compare the previous results with a subperiod begining in 2007Q3. As shown by the lower part of table 3, the contribution of shocks to the variance of the four variables of interest presents some new features. First, the contribution of real shocks is clearly reduced for GDP (only 4.32% for Germany and 7.99% for France instead of 97.51% and 98.02%), while it appears almost negligible for the other variables (most of the time the real

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6See also Smets and Wouters for the US, over the period 2000-2007, they find a very important role of productivity in explaining US business cycles (Smets & Wouters, 2007).

7Similar results can be found in the literature for the euro area (Gerali et al., 2010).
supply shocks account for less than 0.3% of the corresponding variance). In contrast the shock on investment and the financial shocks become the main driving force of activity on this subperiod.

Apart from the increased contribution of financial shocks, the main element to be noticed in the subsample is key role of foreign shocks in explaining the variance of national variables. The international interdependence of macroeconomic variables has clearly increased between Germany and France since 2007Q1: more than 16% of german GDP and more than 20% of french activity are now impacted by the other country shocks. Accordingly, more than 25% of german investment and more than 17% of french investment are impacted by the other country shocks. Turning to financial indicators, more than 28% of fluctuations in the german interest rate loan and more than 23% of fluctuations in the french interest rate loan are explained by the other country shocks. Finally, as already noticed, loan supplies are very sensitive to the cross border diffusion of national shocks. However, one shall now note that the foreign driving force does no longer come from the supply shock (only 0.04 % for Germany and 0.03% for france instead of 13.23% and 7.45% for the whole sample period) but clearly the financial shocks, as it represents respectively 45% for Germany and around 12% for France.

We complete these findings with a comparison of the forecast variance decomposition of output, investment, loan supply and loan interest rates at various horizons based on the mode of the model's posterior distribution in figure 10. The pre-crisis determinants of the GDP movements are similar for Germany and France: in the short run (ie, up to the fourth quarter) movements in GDP are primarily driven by the real supply and demand shocks. As time increases, the weight of the real supply shock becomes predominant to account for almost 100% after 25 years (Q100). In the meanwhile, the contribution of demand and foreign shocks clearly shrinks. The same exercise on the sub sample clearly shows a different picture: in the short run, the weight of supply shocks is clearly reduced, while the contribution of demand and foreign shocks clearly increases. One shall also note that the role of financial shocks is also modified. Indeed, using the whole sample period, financial shocks mainly impact activity in the short run, while, taking into account the sub sample results, the consequence of financial shocks appear to be clearly persistant in the long run (they account for 5% in Germany and 10% in France, at Q100). The sensitivity of national GDPs to the interbank interest rate has also increased in the post crisis sample, as in the long run it still accounts for 10% of the forecast error variance. Thus, the long run contribution of shocks to the variance of GDP offers a different picture. Taking into account the whole sample period, the long run variance of activity only depends on supply shocks, while in the subsample, real supply shocks only account for 50% of, the rest of GDP variance being explained by demand, monetary, financial and foreign shocks.

Regarding Investment, the analysis conducted for the sub sample clearly shows that the financial crisis has induced an increased heterogeneity in investment fluctuations between Germany and France. In the whole sample German investment is mainly determined by supply shocks while financial shocks play a complementary role (around 10% at Q1, 20% at Q4 and 5% at Q100). In the subsample, the weight of supply shocks is the same in the short run while it is clearly reduced in the medium and long run, while financial shocks account for a greater part of the variance (around 25% at Q100). The influence of the interbank interest rate clearly increases in the sub sample period, as the monetary policy shock accounts for around 10% of the forecast error variance from
Q10. Although the situation of the two countries appears somewhat similar in the whole sample, France offers an alternative long run picture in the sub sample: productivity shocks account only for 40%, while financial shocks explain around 50% of the forecast error variance at Q100.

The loan supply variance decomposition clearly offers a very different picture between the whole and the subsample periods. Despite clear differences between Germany and France, in both cases the weight of real shock is clearly reduced in the subsample while the contribution of monetary and foreign shocks have increased. Financial shocks clearly appear to be the main determinant of the forecast variance error in the subsample for all time horizons.

6 Conclusion

In this paper we have developed a two country DSGE model of the European monetary union that takes into account: (1) the bank based nature of finance and (2) the increase in cross banking activities over the last 15 years between members. Given these features we have addressed two main questions: we first evaluated the consequences of introducing cross border lending in the transmission of asymmetric shocks. Under a standard Taylor rule, we found that international bank flows promote more business cycle synchronization. One must also note the increased persistence of shocks on the current account coming from cross border interest payments. Under an extended rule, we found that the main benefits are obtained if the economy is affected by financial shocks, as the extended rule provides an attenuation mechanism in the international transmission of this kind of shocks. In particular, we find that this extended rule promotes a better synchronization in the financial structure of firms. This feature may be interesting since at the monetary union level, monetary policy can only target union wide variables. Introducing interest rate spreads may provide a simple solution to limit to a few countries the consequences of defavourable financial shocks, amplifying the business cycles of these countries, while preserving the economic health of the other members of the monetary union.

The second question we addressed concerns the role of cross border bank lending in the financial crisis of 2007. We performed a historical decomposition of the dynamics of the four main variable of interest (namely activity, investment, loan supply and the interest rate paid by borrowers), by contrasting the whole time period (2003Q1-2012Q3) and a sub sample corresponding to the post 2007 period. We found that, the financial crisis has induced an increased contribution of financial shocks in the explanation of the evolution of the main macroeconomic variables as well as an increase in the sensitivity of output fluctuations to the interbank interest rate. The structure of our model is also helpful to underline some major evolution in the increasing role of foreign shocks as the driving forces of national variables after the financial crisis, as well as an increased heterogeneity in the determination of investment fluctuations between Germany and France.
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A Data sources

- **Gross domestic product**: expenditure approach, millions of national currency, current prices, quarterly levels, seasonally adjusted - sources OCDE stats.

- **Private final consumption expenditure**: millions of national currency, current prices, quarterly levels, seasonally adjusted - sources OCDE stats.

- **Gross fixed capital formation**: millions of national currency, current prices, quarterly levels, seasonally adjusted - sources OCDE stats.

- **Gross domestic product deflator**: expenditure approach, OECD reference year, seasonally adjusted, quarterly - sources OCDE stats.

- **Loans to firms**: total maturity, outstanding amounts at the end of the period (stocks), Euro area (changing composition), Non-Financial corporations (S.11), monthly data (aggregated to get quarterly data) - sources ECB.

- **MFI interest rate**: monthly (taken in average to get quarterly data), Credit and other institutions (MFI except MMFs and central banks); Loans up to 1 year; BS counterpart sector: Non-Financial corporations (S.11); Outstanding amount - sources ECB.

- **Official refinancing operation rates**: central bank interest rates, one year maturity, quarterly data - sources Eurostat.

B Functional forms

B.1 Pareto distribution

Recall that $\omega \sim \mathcal{P}(k; \omega_{\min})$ and $\in [\omega_{\min}; +\infty[$, the profit conditionnal expectation when entrepreneur is creditworthy, $\eta^*\omega = E[\omega|\omega \geq \omega^C] = \int_{\omega^C}^{\infty} \omega f(\omega) d\omega$, while the loss conditionnal expectation is, $(1 - \eta^*)\omega = E[\omega|\omega < \omega^C] = \int_{\omega_{\min}}^{\omega^C} \omega f(\omega) d\omega$. The survival rate is computed as, $\eta^* = \Pr[\omega \geq \omega^C] = \int_{\omega^C}^{\infty} f(\omega) d\omega = (\omega_{\min}/\omega^C)^k$ and death rate is $1 - \eta^*$. The conditional expectations is computed via, $\bar{\omega} = E[\omega|\omega \geq \omega^C] = \frac{\int_{\omega_{\min}}^{\omega^C} \omega f(\omega) d\omega}{\int_{\omega_{\min}}^{\omega^C} f(\omega) d\omega} = \frac{k}{k+1} \omega^C$. Since $E[\omega] = E[\omega|\omega \geq \omega^C] + E[\omega|\omega < \omega^C] = 1, \bar{\omega} = 1 - \bar{\omega}$.
B.2 Investment adjustment costs

The functional form of adjustment cost function is, 
\[ AC_{i,t}^I (I_{i,t}, I_{i,t-1}) = \frac{\chi^I_t}{2} \left( \frac{e^{I_{i,t}} I_{i,t}}{I_{i,t-1}} - 1 \right)^2, \]

when derivating in \( I_{i,t} \),

\[
\frac{\partial [I_{i,t} AC_{i,t}^I]}{\partial I_{i,t}} = \frac{\chi^I_t}{2} \left( 3 \left( \frac{e^{I_{i,t}} I_{i,t}}{I_{i,t-1}} \right)^2 + 1 - 4 \frac{e^{I_{i,t}} I_{i,t}}{I_{i,t-1}} \right)
\]

\[
\frac{\partial [I_{i,t+1} AC_{i,t+1}^I]}{\partial I_{i,t}} = \chi^I_t \left( e^{I_{i,t+1}} \left( \frac{I_{i,t+1}}{I_{i,t}} \right)^2 - \left( \frac{I_{i,t+1}}{I_{i,t}} \right)^3 \right)
\]
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior distributions</th>
<th>Posterior distrib. of $M_1$</th>
<th>Posterior distrib. of $M_2$</th>
<th>Posterior distrib. of $M_3$</th>
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<td>invGam</td>
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Table 2: Prior and Posterior distributions of structural parameters and shock processes
Figure 3: Bayesian response to an estimated positive productivity shock in Germany with financial openness (α^L = 17%) and without (α^L = 0%)
Figure 4: Bayesian response to a negative estimated net wealth shock in Germany with financial openness ($\alpha^L = 17\%$) and without ($\alpha^L = 0\%$)
Figure 5: Bayesian response to an estimated external finance premium shock in Germany with financial openness ($\alpha^L = 17\%$) and without ($\alpha^L = 0\%$)
Figure 6: Bayesian response to an estimated positive productivity shock in Germany with and without spread augmented Taylor rule
Figure 7: Bayesian response to a negative estimated net wealth shock in Germany with and without spread augmented taylor rule
Figure 8: Bayesian response to an estimated external finance premium shock in Germany with and without spread augmented Taylor rule.
Figure 9: Historical Decomposition of Shocks Contributions

Note: The solid blue line depicts the quarterly growth rate in real GDP, Investment and Loans per capita, expressed in percentage point deviations from the model’s steady state. The colored bars depict the estimated contributions of the various groups of shocks (Supply: Productivity and Investment; Demand: Public Spending and Preferences; Financial: External Finance Premium and Net Worth).
Figure 10: Forecast error variance decomposition (at the mode of the posterior distribution) for periods 2003Q1-2012Q3 (left) and post crisis 2007Q3-2012Q3 (right faded). Note: The variance decomposition indicates how much of the forecast error variance of each of Output, Investment and Credit Supply can be explained by exogenous shocks.
<table>
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<td>$\sigma_{ger}^V$</td>
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Table 3: The posterior variance decomposition is the estimated share of variance accounted for by each shock of model $M_2$ and $M_3$.