Simulated Sovereign Credit Events and Their Spillovers to the European Banking System - The Interplay Between Sovereign Bond and CDS Holdings [*]

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Abstract

This paper models and simulates sovereign credit events and their spillovers to the European banking system, focusing on the interplay between banks' sovereign bond and CDS holdings. The incorporation of the CDS market into our theoretical framework enables analysing credit risk transfer mechanisms. Moreover, the model includes features of market and liquidity risk and allows for contagious propagation of counterparty failures. The model is calibrated using public data on 65 major European banks from EBA EU 2011 Capital Exercise. We find that banks' losses due to direct and correlated sovereign bond exposures are significantly higher than the pure losses due to sovereign CDS exposures and to counterparty risk on the CDS market. The main risk for CDS sellers is found to be sudden increases in collateral requirements on multiple correlated CDS exposures. Risk-mitigation mechanisms, including collateralization, collateral netting agreements and close-out netting considerably reduce the extent to which contagion may occur.

J.E.L. Codes : G21, H63, G15.

Keywords : sovereign credit events, credit default swaps, contagion, collateral, market risk, liquidity risk.

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Non-technical summary

This paper models and simulates sovereign credit events and their spillovers to European banks, focusing on the interplay between banks' sovereign bond and CDS holdings. It provides a theoretical framework to assess the potentially risk-mitigating or risk-amplifying role of the CDS market in case of a simulated sovereign credit event. Five transmission channels from sovereign entities to banks are featured : *(i)* direct losses on sovereign bond holdings ; *(ii)* write-downs on other (available for sale and held for trading) sovereign exposures ; *(iii)* direct CDS repayments triggered by the simulated credit event ; *(iv)* increased collateral requirements to cope with higher CDS spreads on other non-defaulted reference entities ; *(v)* contagious propagation of counterparty failures. The incorporation of the CDS market into our theoretical framework enables analysing credit risk transfer mechanisms. Moreover, one contribution of the model is that it explicitly incorporates several features proper to OTC derivatives markets, including collateralization, collateral netting agreements and close-out netting procedures in case of counterparty default.

The theoretical framework is calibrated using public data released by the European Banking Authority (EBA) on 65 major European banks related to the EU 2011 Capital Exercise. The dataset includes both sovereign bond and CDS holdings at a bank level for 28 European sovereign entities, while bilateral CDS exposures are estimated and their market values simulated. We study simulated exogenous sovereign credit event scenarios for a wide range of recovery rates.

We find several interesting results. First, the results show that, given a simulated sovereign credit event, banks' losses due to direct and correlated sovereign bond exposures are significantly higher than the pure losses due to sovereign CDS exposures and to counterparty risk on the CDS market, even though the relative share of each failure channel depends on the recovery rate. Given the home bias on banks' portfolios, losses on direct sovereign exposures are found to be more substantial for domestic banks, whereas losses through correlated sovereign bond exposures are found to be more important for foreign banks. Second, CDS repayments are overall found to remain small compared to banks' capital or liquid assets. Instead, the main risk for CDS sellers is found to be sudden increases in collateral to be posted on multiple correlated exposures. This channel dominates when the recovery rate is high enough, and is more important if the pool of available collateral is correlated with the bond exposure experiencing a credit event. Third, we are not able to document redistributive effects of net CDS repayments in case of a simulated credit event, neither from banks with low exposure to highly-exposed banks nor from highly-liquid banks to banks with lower liquidity. Even though the observed distribution of net protection bought through CDS does not match the distribution of underlying sovereign bond holdings, we do not find significant failures due to the inability of some banks to honour their contractual repayments in case of credit event. Fourth, contagion purely due to CDS exposures is of very limited extent. Potential explanations include the effectiveness of collateral management schemes, the fact that none of the major dealers is found to fail, and that several types of interconnections between banks (interbank loans and deposits, other derivatives) are not accounted for. Fifth, the effectiveness of risk-mitigation mechanisms, including collateralization, collateral netting agreements and close-out netting is analysed. Collateral netting agreements increase banks' liquidity. Close-out netting in case of counterparty default is found to reduce the extent of contagion.

1 Introduction

This paper models and simulates sovereign credit events and their spillovers to European banks, focusing particularly on the interplay between their sovereign bond and credit default swap (CDS) holdings. Sovereign credit events have been a growing concern in recent years, due to the European sovereign debt crisis and to the Greek debt restructuring in March 2012. Even though sovereign defaults have been studied both from a theoretical (see Adam and Grill [2011]) and from an empirical perspective (see Das et al. [2012]), so far no encompassing model has been proposed and estimated to assess the various consequences of a sovereign credit event on a financial system where banks are linked through their CDS exposures.

The contribution of the paper in this regard is twofold. First, from a theoretical perspective, we propose a framework that captures the consequences and spillovers of a simulated sovereign credit event to a system of interconnected banks. Conditional on a credit event, our theoretical framework features several channels through which bank failures may occur and spread : *(i)* direct losses on sovereign bond holdings, *(ii)* write-downs on other correlated sovereign exposures, *(iii)* CDS repayments triggered by the credit event, *(iv)* increased collateral requirements to cope with higher CDS spreads on non-defaulted reference entities and *(v)* contagious propagation of counterparty failures.

Second, from an empirical perspective, we simulate European sovereign credit event scenarios on a sample of 65 large European banking groups using public data from the European Banking Authority (EBA) EU 2011 Capital Exercise. The dataset contains information not only on banks' sovereign bond and corresponding CDS holdings, but also on their balance sheet characteristics. The data is available as of end-September 2011, i.e. the height of the sovereign debt crisis. The study of a sovereign credit event and its interaction with the CDS market at this given point in time is therefore particularly relevant and, to our knowledge, comparable estimates have not yet been computed. In the analysis, we assess the relative importance of the above five different risk propagation channels for a wide range of recovery rates. Whereas bank failures due to losses on bond exposures are important when the recovery late is low, the share of failures due to collateral shortage increases with the recovery rate. The results highlight the importance of second-round effects, i.e. price effects on bonds and CDS other than the entity experiencing the credit event. In particular, write-downs on correlated sovereign bonds exposures are shown to induce large capital losses for most banks. These results are obtained for simulated sovereign credit events occurring one by one. The dynamics of joint sovereign credit events is left for future work.

One key focus of the paper is on the interplay between banks' sovereign bond and CDS holdings within the context of a simulated sovereign credit event. Whereas CDS are used for hedging purposes on a day-to-day basis, the existing literature does not provide an answer about whether the CDS market as a whole plays, *in a generally distressed environment*, a risk-mitigating or a risk-amplifying role. In this regard, the theoretical framework and its empirical estimation enable us to address two main concerns expressed about the potential fragility of the CDS market. First, concerns related to the ability of the market to settle a major credit event, which mainly stem from the large notional CDS amounts at stake (21.8 trillion euros at a global level in December 2011, according to the BIS). Despite the importance of gross notional amounts, we find net CDS repayments from a bank to a bank following a credit event to be relatively low compared to banks' capital and liquid assets. Such finding gives ground to the observation by Coudert and Gex [2010] about the historical resiliency of the CDS market at times defaults with large open interest had to be settled. By contrast, we show that the main vulnerability inherent to the CDS market is the potential inability of one or several CDS sellers to post collateral on multiple correlated CDS exposures whose spreads increase simultaneously. Besides, the results do not show significant redistributive effects of CDS in case of simulated sovereign credit event.

Second, we address concerns regarding counterparty risk and the potential for contagion. A comprehensive survey on this issue has been provided by ECB [2009]. A major observation concerning the CDS market is that most institutions are both gross buyers and sellers of protection, therefore relying on receivables from third parties to honor their own repayments in case of a reference entity default. Our results show little contagion due to the imputation of counterparty failures, partly due to the fact that failing banks are not the major dealers on the CDS market. Whereas the effect of collateralization is relatively limited in the present empirical estimation, close-out netting mechanisms are found to reduce considerably the extent of potential contagion. In particular, it is shown to reduce significantly losses induced by the resolution of a failing bank's derivatives portfolio. Another explanation for the limited extent of contagion in our empirical application is that, when simulating credit events, we do not observe the failure of at least one of the main dealers. Thus, we are only able to study counterparty risk stemming from small or medium-size failing banks, whose participation on the CDS market is limited.

Potentially due to data restrictions, very few comparable papers exist in the literature [4] Whereas the literature on sovereign-bank spillovers is growing

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[4] There exists, however, a large literature (recently surveyed by Upper [2011] on contagion in the interbank market. One important result from this research stream is that contagion through direct linkages is often limited, whereas widespread contagion is likely to occur only when common shocks or price contagion mechanisms are included. See also the paper by Glasserman and Young [2013] who study contagion in the interbank market

(significant papers include Alter and Schuler [2012] and Ejsing and Lemke [2011]), the interplay between sovereign bonds and CDS holdings in a distressed environment when counterparty risk exists has not been comprehensively assessed earlier. Two models of defaults on the CDS market include Heise and Kuhn [2012] and Markose et al. [2012]. Heise and Kuhn [2012] propose a stochastic model in which financial institutions are interconnected through the CDS market. The latter can amplify contagion rather than mitigate it, due to the fact that CDS are primarily used to expand banks' loan books as they are thought to offload additional credit risk from the balance sheets. Markose et al. [2012] study the centrality of the main market participants and their potential "super-spreader" role in a network structure. Their results, largely based on data simulated from aggregate FDIC fill-ins, suggest that a set of institutions concentrating a large share of the activity, or "too interconnected to fail" should be taxed based on their centrality.

Our theoretical framework differs from those proposed in Heise and Kuhn [2012] and Markose et al. [2012] or other earlier studies in several aspects. First, we do not focus on CDS exposures in isolation, but rather on the interplay between banks' sovereign CDS and bond holdings. Considering derivatives exposures without considering the portfolio of underlying credit exposures might lead to biased results, as one cannot then observe whether CDS are used for hedging truly held underlying positions, for macro hedging or for speculating. Second, our theoretical framework captures several features inherent to derivatives markets that have been largely ignored in the academic literature up to now. This includes the widespread practice of collateralization and variation margins, as well as close-out netting in case of a counterparty failure. Our model is flexible and can accommodate different assumptions on banks' behaviour, especially as far as collateral management (computation of margin requirements and pledgeable assets, rehypothecation and collateral netting) is concerned. Collateral management practices are assessed, but their full exploration is left for future work. Thus, rather than focusing on only one channel through which contagion following a credit event may occur, as a third contribution, our theoretical framework captures several channels of different nature, i.e. factors related to idiosyncratic and system-wide risks (including the correlation of exposures) as well as to solvency or liquidity risks.

The rest of the paper is structured as follows. Section 2 presents the theoretical framework. Section 3 describes the dataset, while Section 4 exposes the calibration. Section 5 shows several scenarios of sovereign credit events and simulation results. Section 6 explores the dynamics of the model with alternative collateral management schemes. Most tables and figures are presented in the appendices.

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using data from EBA 2011 Stress Tests.

2 The theoretical framework

2.1 Timeline

There is a set $\Omega = \{1, ..., n\}$ of financial institutions indexed by i, also refered to as banks. Each of them is given a stylized balance sheet representation presented on figure 1. Each institution i has a portfolio of sovereign bonds along with other assets. There is a set $\Theta = \{1, ..., J\}$ of sovereign entities[5] indexed by j. The holdings by bank i of bonds emitted by country j are denoted B_{ij}. In addition, each institution can be both gross and net buyer or seller of sovereign CDS, which enter the balance sheet at fair value. The gross CDS notional sold by bank i to bank k on the reference entity j is denoted g_{ik}^{j}. The assets are financed both with equity K_i (also called "capital") and other liabilities.

Assets	**Liabilities and equity**
Sovereign bonds $\sum_j B_{ij}$	Equity K_i
CDS bought	CDS sold
Other assets	Other liabilities

FIGURE 1 – **Stylized balance sheet of bank i.**

The timeline of the theoretical framework features an initial exogenous credit event. The sequence of events we simulate, which includes both direct and indirect (or feedback) effects of a sovereign credit event on financial institutions' balance sheets and interconnections, goes as follows :

- **(1)** A sovereign entity defaults on part or all of its sovereign debt. A recovery rate is observed and corresponding direct losses are imputed on banks' capital.
- **(2)** The tail dependences of other sovereign bonds with the defaulted sovereign's bonds are estimated. Banks' other sovereign bond holdings (available for sale and held for trading) are marked to market, therefore written down.
- **(3)** The tail dependences of the CDS spreads of the other sovereigns with the CDS of the defaulted sovereign is estimated. CDS protection sellers are required to post more collateral to face overall higher sovereign CDS spreads, while the pledgeable value of their bond holdings is lower. If they cannot meet the collateral requirement, they fail from collateral shortage.

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[5] Given the importance of sovereign bonds in banks' balance sheets, we focus on this asset class. The same theoretical framework could nevertheless be applied for the study of other credit events, such as corporate credit events.

- **(4)** CDS payments are triggered on the country that first experienced a credit event.
- **(5)** Banks failing at stages **(1)**, **(2)** and **(3)** are not able to honor their CDS payments. Their derivatives contracts with other banks are terminated for both the defaulted sovereign entity and for all other reference entities (close-out netting), potentially leading to additional losses for their counterparties.
- **(6)** Losses from counterparty to counterparty are imputed until no more bank fails.

Let us now expose each of these building blocks in more extensive details.

2.2 Imputation of losses

Two types of losses are induced by the initial default of a reference entity. First, any sovereign holding that cannot be recovered is imputed negatively on the capital stock. Let us denote $\bar{j} \in \Theta$ any initially defaulting reference entity and $RR_{\bar{j}}$ the recovery rate on this particular bond. A *scenario* is fully defined by $\{\bar{j}, RR_{\bar{j}}\}$. If K_i denotes the capital of a particular bank i, it then incurs a direct loss $B_{i\bar{j}}(1 - RR_{\bar{j}})$ as a consequence of the default of $\bar{j}$. It is insolvent if :

$$K_i - B_{i\bar{j}}(1 - RR_{\bar{j}}) \leq 0 \tag{1}$$

In addition to direct losses, our theoretical framework features indirect losses on other sovereign bond holdings. More precisely, there is ample evidence for the co-movement of similar asset classes in response to a shock affecting only one of them in the first place. In our particular case, a credit event on one sovereign entity might give rise to contagion to other European sovereign entities. Such spillovers have been extensively documented in the recent literature (see Gande and Parsley [2005] for example).

For any initially defaulting reference entity $\bar{j}$, we compute the tail dependence of the market price of bond $B_{\bar{j}}$ with all other bonds B_j for j in $\Theta_{-\bar{j}}$ (i.e. the set of all sovereign entities expect $\bar{j}$). Let us denote p_j the market price of a one unit sovereign bond j. To estimate the change of other bonds prices induced by a default of $\bar{j}$, we assume a jump of $p_{\bar{j}}$ at the time of the simulation to $p_{\bar{j}}^{RR}$, the price implied by the assumed recovery rate (if there is no arbitrage possibility). Assuming the recovery value is paid immediately, the market price of a defaulted bond with a 1 euro face value must exactly equal the recovery rate. Thus, $p_{\bar{j}}^{RR} = RR_{\bar{j}}$.

The estimation of tail dependences gives rise to a vast econometric literature. Widely used methods include quantile regressions or copula models. In this paper, the price change of bonds $j \neq \bar{j}$ in response to a jump of $\bar{j}$ to its recovery value is estimated using a copula framework. Given our later focus on jumps to default, the t-Copula (described in Demarta and McNeil [2005]) is chosen for its ability to account for statistically extreme events. Several papers have shown that the empirical fit of the t copula is generally superior to that of the Gaussian copula for modeling financial returns (see Mashal and Zeevi [2002] or Breymann et al. [2003]). The bivariate copula density between the prices of bonds j and k is given by :

$$C\left(u_j, u_k\right) = F\left(F_j^{-1}\left(u_j\right), F_k^{-1}\left(u_k\right)\right) \tag{2}$$

, where F_j^{-1} and F_k^{-1} are the quantile functions of the margins. The parameters for the copula that best fit the data are obtained by maximum likelihood. Given a drop $\epsilon_{\bar{j}} = p_{\bar{j}} - p_{\bar{j}}^{RR}$ in the price of bond $\bar{j}$, the correlated drop (or eventually rise) of the price of any bond j at the quantile κ, denoted ϵ_j, is obtained according to :

$$\epsilon_j = \rho_{\bar{j}j}\epsilon_{\bar{j}} + \sqrt{1 - \left(\rho_{\bar{j}j}\right)^2} F^{-1}\left(\kappa\right) \tag{3}$$

Conditional on a scenario on bond $\bar{j}$, one such ϵ_j is estimated for all other sovereign bonds $j \in \Theta_{-\bar{j}}$. For a bank i holding B_{ij}, the write-down on the particular position is $\epsilon_j \alpha_{i,j} B_{ij}$, where $\alpha_{i,j}$ is the share of its bond exposure to j that is either available for sale or held for trading (therefore marked-to-market). The remaining share $\left(1 - \alpha_{i,j}\right) B_{ij}$, held-to-maturity, is not marked-to-market according to the prevailing accounting standards, and is therefore not assumed to suffer from any immediate write-down. After the imputation of direct and correlated losses from sovereign bond exposures, a bank is *insolvent* if :

$$K_i - B_{i\bar{j}}(1 - RR_{\bar{j}}) - \sum_{j \in \Theta_{-\bar{j}}} \epsilon_j \alpha_{i,j} B_{ij} \leq 0 \tag{4}$$

2.3 CDS holdings and collateral requirements

CDS are now introduced into the framework. Depending on their observed distribution among banks, credit default swaps may either play a mitigating or an amplifying role in the case of sovereign credit event. Assuming credit risk protection through CDS is mainly sold by banks with little sovereign exposure and bought by heavily exposed institutions, they may absorb a large part of the consequences of a sovereign credit event. On the contrary, if net protection is sold by banks heavily exposed to sovereign risk, or if the network of bilateral CDS exposures is so dense that the inability of one institution to honour its contractual obligations might entail a cascade of failures, CDS markets might then play an amplifying role. The present subsection introduces CDS holdings and corresponding collateral requirements. In particular, it estimates additional collateral requirements on reference entities $j \in \Theta_{-\bar{j}}$ needed to cope with a failure of $\bar{j}$. The next subsection proposes an explicit modelling of counterparty failure and contagion, and solves for the end-number of bank failures and payments to be honoured.

For each reference entity $j \in \Theta$, there exists a $n \times n$ matrix of bilateral gross notional CDS sold on the reference entity j. Each of its components g_{ik}^j is the gross notional sold by bank i to bank k on the reference entity j. $g_{ii}^j = 0$ must hold for all i, i.e. no bank can sell CDS to itself. The matrix of net protection sold, whose components are denoted n_{ik}^j is given by :

$$n_{ik}^j = \begin{cases} 0 & \text{if} \qquad g_{ki}^j > g_{ki}^j \\ g_{ik}^j - g_{ki}^j & \text{otherwise.} \end{cases} \tag{5}$$

Each CDS transaction in our framework is assumed to be collateralized. According to the ISDA, well-above 90% of the transactions on sovereign CDS are collateralized [ISDA, 2012b]. The fact that a transaction is collateralized, however, does not mean that it is *fully* collateralized. In conformity with a widespread market practice, a CDS position is assumed to be fully collateralized if the amount of collateral required to be posted by the selling institution i to the buying institution k is *(i)* the market value of the contract in case it is negative for i or *(ii)* zero if it is positive for i. Partial collateralization is nevertheless a current market practice. Only a fraction τ_{ik}^j of any deal between i and k on a CDS on reference entity j is assumed to be collateralised. τ is positive and here constrained to be below one, i.e. we do not account for the possibility of over-collateralisation. Furthermore τ is not market-specific but exposure-specific so as to account for the diversity of market practices and of differences in perceived counterparty risk. It is assumed to be fixed by contract, so that it does not increase with the CDS spread q^j. Finally, we assume reciprocity in bilateral transactions, i.e. $\tau_{ik}^j = \tau_{ki}^j$.

On the market, both buyers and sellers of CDS usually have to post collateral depending on the market value of any bilateral deal. Collateral calls, nevertheless, might be a major risk for the protection sellers only, as the collateral required for them to be posted may surge quickly if the spread of the underlying reference entity rises sharply or jumps to default. In contrast, the buyer of a CDS contract has only committed to pay a quarterly fixed premium that does not vary with the CDS spreads, and which usually represents only a small fraction of the notional amounts insured. In the following, only collateral posting by protection sellers is considered.

Denote $V_{ik,t+h}^j\left(\lambda_{t+h}^j, q_t^j\right)$ the market value at date $t+h$ of a CDS contract signed at date t between counterparties i and k on reference entity j. From the buyer's perspective, it is the difference between the present value of the default-contingent payment and that of the future stream of premia. It depends crucially on the agreed-upon premia q_t^j to be paid annually by the protection buyer per unit of notional amount and of the prevailing default intensity λ^j_{t+h}. In the following, $V^j_{ik,t+h}(\bullet)$ consistently denotes the market value for the *buyer* i (from seller k) of a CDS on the reference entity j. The value for the seller is given by $V^j_{ki,t+h}(\bullet) = -V^j_{ik,t+h}(\bullet)$.

Any bilateral exposure n^j_{ik} between any i and k may result from several offsetting or reinforcing transactions as it is common on the CDS market, each of them contracted at a different point in time and having a different present market value. The net market value for i of a bilateral exposure with k on reference entity j is given by the sum of all positive and negative market values of the non-matured transactions performed in the past and is denoted $\tilde{V}^j_{ik,t+h}$. In other terms :

$$\tilde{V}^j_{ik,t+h} = \sum_{v<h} V^j_{ik,t+h-v} \tag{6}$$

The amount of collateral to be posted at some date t by any institution i to any institution $k \neq i$ on any reference entity j, denoted c^j_{ik} is then :

$$c^j_{ik} = \begin{cases} 0 & \text{if } \tilde{V}^j_{ik,t+h}\left(\lambda^j_{t+h}, q^j_t\right) < 0 \\ \tau^j_{ik}\tilde{V}^j_{ik,t+h} & \text{if } \tilde{V}^j_{ik,t+h}\left(\lambda^j_{t+h}, q^j_t\right) > 0 \end{cases} \tag{7}$$

The amount c^j_{ik} corresponds only to what is usually called *variation margins*, i.e. collateral posted as a response to changes in the market value of a contract. Other types of margins, especially initial margins, are not modelled in the present framework, as the extent to which they prevail in non-centrally cleared bilateral transactions is not well documented.

If there were no collateral netting agreements across reference entities - i.e. if both parties were to post collateral to one another - the total amount of collateral posted by any bank i to $k \neq i$ would be equal to $\hat{c}_{ik} = \sum_j \mathbf{1}_{\left\{\tilde{V}^j_{ki,t+h}(\bullet)>0\right\}} \tau^j_{ik}\tilde{V}^j_{ki,t+h}(\bullet)$, where $\mathbf{1}_{\{\bullet\}}$ denotes the indicator function. Nevertheless collateral netting agreements became very popular among financial institutions after 2000 as documented by ISDA [2012b]. In the current framework, we assume collateral netting between all CDS on reference entities in Θ. With collateral netting, the total amount of collateral to be posted by i to k on all CDS trades is :

$$\begin{aligned} c_{ik} &= \max\left\{0, \sum_{j=1}^{J} \tau^j_{ik}\left[\mathbf{1}_{\left\{\tilde{V}^j_{ki,t+h}>0\right\}}\tilde{V}^j_{ki,t+h} - \mathbf{1}_{\left\{\tilde{V}^j_{ik,t+h}>0\right\}}\tilde{V}^j_{ik,t+h}\right]\right\} && (8) \\ &= \max\left\{0, \hat{c}_{ik} - \hat{c}_{ki}\right\} && (9) \end{aligned}$$

This implies that in any combination of two banks, only one is pledging collateral *vis-a-vis* the other. Finally, the total amount of collateral to be posted by bank i at any time to all counterparties and for all reference entities is equal to :

$$c_i = \sum_{k \in \Omega \setminus \{i\}} c_{ik} \tag{10}$$

Once collateral is modelled, what is essential is the change in collateral requirements induced by an exogenous change. Denote $\bar{C}_i$ the pledgeable value of the assets A_i of institution i. According to ISDA [2012b], more that 90% of collaterals on OTC markets are cash and sovereign bonds. The pledgeable value of a sovereign bond is typically smaller than its face value, as an haircut is applied. No haircut applies on cash collateral. The pledgeable value of the assets of institution i is :

$$\bar{C}_i = \sum_m a_{mi} \left(1 - h_m\right) \tag{11}$$

, where $h_m \in [0;1]$ is the haircut demanded on the m-th asset a_m, and where we ensure that $\sum_m a_{mi} = A_i$ for all i. Thereafter the pledgeable value of bond holdings will be affected by sovereign credit events. Importantly, we do not include collateral received in $\bar{C}_i$, i.e. we do not allow for rehypothecation.

In this framework, a bank is said to be liquid at some point in time if it is able to meet its margin calls. In contrast, a bank is *illiquid* and fails from *collateral shortage* if :

$$c_i > \bar{C}_i \tag{12}$$

Such definition of illiquidity is, of course, narrow as it ignores other funding issues that lie outside the scope of this model. Nonetheless, the difference between failures from *insolvency* - the value of an institution's assets falling below the value of its liabilities - and failures from *illiquidity* - the inability to meet collateral calls - is important to highlight, as the policy implications arising from one or another are of different nature. We are later able to provide empirical evidence on the relative magnitude of both types of failures.

At this stage, we add two joint mechanisms that possibly contribute to the spread of the contagion once the default of a reference entity $\bar{j}$ occurs. First, the spreads of all CDS on reference entities $j \in \Theta_{-\bar{j}}$ might increase, thereby increasing collateral requirements on net protection sold on all - or on a subset of - non-defaulted reference entities. Second, the value of the pledgeable collateral decreases. In this paper, we assume haircuts to be fixed on all asset classes, and model instead the decreasing value of the assets on which these haircuts are applied.

First, one needs to estimate the increase in collateral requirements induced by a particular sovereign event. Recall that the market value of a position (then the collateral a bank has to post) depends crucially on the price q^j. Therefore, to estimate $\Delta\tilde{V}^j_{ki,t+h}(\bullet)$, one needs estimates of $\left(\Delta q^j|\Delta q^{\bar{j}}\right)$ for all $j \in \Theta_{-\bar{j}}$, i.e. the change of the price of a CDS of the reference entity j conditional on $\bar{j}$ jumping to default. The magnitude of $\Delta q^{\bar{j}}$ - the difference between the last spread observation and the spread at default -, which depends on the recovery rate RR_j is discussed in the section on calibration.

The tail dependences between CDS prices on reference entities $\bar{j}$ and j are estimated using the same copula framework as the one described above (equations 2 and 3). We assume a Student t distribution for the margins of the CDS spreads in first-differences. Given a price rise $\Delta q_{\bar{j}}$, the response of the CDS spread on reference entity j is given by :

$$\Delta q^j = \Delta q^{\bar{j}}\rho^{CDS}_{\bar{j}j} + \sqrt{1-\left(\rho^{CDS}_{\bar{j}j}\right)^2}F^{-1}(\kappa) \tag{13}$$

, where $\rho^{CDS}_{\bar{j}j\_}$ is the correlation between the spreads of CDS on reference entities j and $\bar{j}$. The total increase in collateral to be posted by any bank i is given by :

$$\Delta c_i = \sum_{k\in\Omega\backslash\{i\}} \max\left\{0, \sum_{j=1}^{J}\Delta\tilde{V}^j_{ik,t+h}(\bullet, \Delta q_j)\,\tau^j_{ik}\left[\mathbf{1}_{\left\{\tilde{V}^j_{ki,t+h}>0\right\}} - \mathbf{1}_{\left\{\tilde{V}^j_{ik,t+h}>0\right\}}\right]\right\} \tag{14}$$

Parallel to this increased collateral requirement, the value of the pledgeable assets falls as a result of direct losses and write-downs on sovereign bonds. The value of the collateral drops from $\bar{C}_i$ to :

$$\bar{C}'_i = \bar{C}_i - B_{i\bar{j}}\left(1-RR_{\bar{j}}\right) - \sum_{j\in\Theta_{-\bar{j}}} B_{ij}\left[1-\epsilon_j\alpha_{i,j}\right]\left(1-h_j\right) \tag{15}$$

An important feature of equation 15 is that the decrease in the value of the collateral pool depends on the asset composition of this pool. Therefore, if the collateral pool of a bank i is composed to a large extent of the sovereign bond $\bar{j}$ or of other bonds highly correlated with $\bar{j}$, then it might shrink considerably as a result of particular sovereign default scenarios.

For a bank i to be liquid once the increased collateral requirements and the shrinkage of the pool of pledgeable assets are accounted for, the following condition needs to hold :

$$\bar{C}'_i \geq c_i + \Delta c_i \tag{16}$$

In case condition 16 does not hold, bank i fails.

2.4 CDS repayments and counterparty risk

If the event leading to write-downs on sovereign bonds and to CDS spreads increases is to be classified as a *credit event*, then CDS repayments have to be honoured. CDS repayments are assumed to be paid out of the pool of cash and liquid assets that is also used to post collateral, i.e. $\bar{C}_i'$. If no bank failed, then any bank i is able to honour its scheduled CDS repayments if :

$$\bar{C}_i' - (c_i + \Delta c_i) + \sum_k n_{ki}^{\bar{j}} \geq \sum_k n_{ik}^{\bar{j}} - \sum_k c_{ik}^{\bar{j}} \tag{17}$$

The left-hand side of equation 17 corresponds to the amount of cash and liquid assets that has not yet been pledged as collateral plus the net repayments to be expected from all counterparties k. Its right-hand side corresponds to the sum of what has to be paid less the collateral that has already been posted on those positions to cover for the increased credit risk. If condition 17 does not hold for bank i, it fails from *contagious illiquidity.*

Until now, no counterparty risk has been accounted for in this framework. Nonetheless, an important feature of our theoretical framework is that it accounts for potential counterparty failure by replicating several real-world features inherent to derivatives market. When studying the CDS market, taking counterparty risk into account is of particular importance given the substantial difference between gross and net notional outstandings. If one restricts attention to net protection bought or sold (therefore assuming implicitly that all payments are made in full), then failures due to CDS payments are unlikely to occur, if the amounts considered are overall of small magnitude when compared with banks' capital buffers, as is the case with the 65 European banks whose balance sheet has been disclosed by the EBA. On the contrary, if each institution is relying on repayments from other institutions to make its own payments, then one bank's failure to pay within the whole chain of obligations might entail a cascade of contagious bank failures. The last part of our theoretical framework accounts for such possibility.

To model counterparty risk and the potential for contagious failures, we use a sequential procedure that accounts for the specificities of the derivatives market, in particular the *close-out netting* of all derivatives' deals in case of a failure. Close-out netting refers to the termination procedure of all derivatives transactions concluded under a given contract, usually the ISDA Master Agreement. Three steps are involved : *(i)* the termination of all obligations contracted between a failing and a non-failing party, *(ii)* the calculation of the replacement value of each of the deals, and *(iii)* the summation of all positive and negative replacement values in order to derive a single net payable or receivable. A clear description of the functionning of close-out netting can be found in Mengle [2010]. The *ex post* advantages[6] of close-out netting for risk mitigation are clear. If each transaction were to be considered as a separate contract, the non-failing party would have to repay immediately all the replacement values of its out-of-the-money derivatives deals with the failing party, and then to wait for months before receiving some part of its ongoing transactions that were in-the-money. The difference is considerable, given that gross amounts are typically several times higher than net amounts. For a proper modelling of counterparty risk, such feature has to be accounted for.

Another feature of our framework is that, during the resolution procedure, banks might fail from *contagious failures* (by contrast with fundamental failures) through either a solvency channel or a liquidity channel. The *contagious insolvency* channel comes from the losses due to counterparty failures that are imputed on the capital stock of a bank and may drive it below zero. The *contagious illiquidity* channel is due to a bank being unable to deliver its own CDS repayments in case some of its counterparties do not repay them. The existence of these two channels implies that, when the default imputation procedure stops, all non-failed banks *(i)* have positive equity and *(ii)* have been able to honour all repayments imposed by the resolution scheme. We must mention here that our theoretical framework does not feature contagion through other interbank exposures, either loans and borrowings on the money market or other forms of exposures.

At this stage, the number of what we call *fundamental failures* is given as a result of the implementation of the first part of the theoretical framework (equations 4 and 16 in particular). The set of failed banks is the union of the two sets of insolvent and illiquid institutions. After imputing the effects of a common sovereign shock on individual banks, we now impute the failure of particular banks on their direct counterparties in the network. When a bank k fails as a consequence of a sovereign default $\bar{j}$, the losses for each of its counterparties $i \in \Omega \backslash \{k\}$ are twofold. First, contingent payments due to the failure of $\bar{j}$ cannot be honoured. Second, all other derivatives contracts, including CDS on all reference entities other than $\bar{j}$, are terminated.

Given the existence of a close-out netting mechanism, those two losses can be considered at once. What matters is not the termination of the particular contract between i and k on reference entity $\bar{j}$, but the termination of all CDS contracts at the same time. Regarding the hedging role of collateral received, one must therefore consider collateral posted not on particular positions but collateral pooled across all positions between two institutions. For all i, k and j, denote $\tilde{V}^j_{ik,def} = \tilde{V}^j_{ik,t+h} + \Delta\tilde{V}^j_{ik,t+h}$ the market value of a transaction once

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[6] *Ex ante*, some other issues needs to be raised. It might be that the very possibility of close-out netting in case of failure induces, in the first place, all counterparties to engage in an greater number of transactions, potentially leading to overall higher systemic risk levels. We do not discuss such issue in the present article.

the default of $\bar{j}$ occured. A particular case is that of $\tilde{V}^{\bar{j}}_{ik,def}$, which equals the CDS repayments that have to take place between i and k, and which depends on the recovery rate $RR_{\bar{j}}$ on the sovereign. Whether the failure of k creates a liability of k vis-a-vis i or the contrary is determined by the sign of $\tilde{V}_{ik,def} = \sum_j \tilde{V}^j_{ik,def}$. Two cases may arise. If $\sum_j \tilde{V}^j_{ik,def} < 0$, i.e. if the market value of all derivatives positions between i and k is negative for the non-failed party i, then it has to repay $\sum_j \tilde{V}^j_{ki,def}$ to k. The fact that i's counterparty failed does not change anything to the execution of the contract in that regard. This last amount, however, must not be considered as a loss ; it is a payment made in the due execution of a contract. It is assumed to be paid with available cash or with highly liquid securities similar to those used as collateral. A fraction τ_{ik} of this position being already collateralized, bank i must repay only a share $1 - \tau_{ik}$ of this net payable, assuming $\tau_{ik} < 1$[7]. If it is not able to do so, it fails from *contagious illiquidity.*

In the second case, where $\sum_j \tilde{V}^j_{ik,def} > 0$, then the failed party k has a liability *vis-a-vis* i that it cannot honour in full. Nevertheless i can recover the collateral that k posted before the jump-to-default. In addition, the whole recovery rate on the uncollateralized part of the exposure is determined by the complex structure of liabilities' seniority. It is here assumed to be exogeneously given and equal to RR_k. The overall recovery value for i is then given by :

$$\min\left\{1, \frac{\bar{C}'_k}{c_k + \Delta c_k}\right\} c_{ik} + (1 - \tau_{ik})\, RR_k \sum_j \tilde{V}^j_{ik,def} \tag{18}$$

The first term in equation 18 corresponds to the collateral received by i from k. In case k was liquid but failed from insolvency, it delivered all collateral due, i.e. c_{ik}. On the contrary, if it failed for insolvency, it was only able to deliver a fraction $\bar{C}'_k / (c_k + \Delta c_k)$ of the collateral c_{ik} it was supposed to post. The second term corresponds to the uncollateralized part of the counterparty risk on which a counterparty-specific recovery rate is applied.

The total loss for counterparty i due to the termination of all its CDS contracts with k is equal to

$$(1 - \tau_{ik})\,(1 - RR_k) \sum_j \tilde{V}^j_{ik,def} \tag{19}$$

This loss is imputed on K_i, the capital of institution i, and for all failing counterparties k. If it is large enough and drives K_i below zero, then i fails from *contagious insolvency.*

The sequential procedure for the imputation of counterparty failures then works as follows :

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[7] If $\tau_{ik} > 1$, then i actually receives back a portion $(\tau_{ik} - 1)$ of the collateral it posted.

- **(1)** The set of institutions failing conditional on a scenario is known *ex ante*. Losses due to partial CDS repayments and termination of other contracts are imputed to all non-failing institutions. Their net value (i.e. the net value of their equity) is computed.
- **(2)** If all net values are positive, the procedure stops. On the contrary, all institutions for which $K_i < 0$ fail from contagious insolvency. For failed institutions, an endogenously determined recovery rate is computed (the amount of their scheduled repayments that they will be able to honour).
- **(3)** All institutions need to honour their CDS repayments, either in full (if non-failed) or partially (if failed). If a not-yet-failed institution does not hold enough cash and liquid assets to repay for the protection it sold (equation 17), it fails from contagious illiquidity.

Failures at stages **(2)** and **(3)** might entail losses for other institutions. All CDS repayments due to the failure of $\bar{j}$ have been settled, partially or in full, during stage **(3)** of this first iteration. The only losses that can be imputed at this stage are those linked to the termination of other derivatives contracts with positive value. We iterate on the previously described steps.

- **(4)** All institutions failling at stage **(2)** and **(3)** terminate all their CDS contracts. These losses are imputed on the capital of the smaller set of non-failed institutions.
- **(5)** Iterate stage **(4)**. The procedure stops either when all banks are bankrupted or when all non-failed institutions have a positive equity value after imputation of all losses due to the failure of their counterparties.

3 The dataset

Our main dataset has been released by the European Banking Authority (EBA) in December 2011 as part of its EU 2011 Capital Exercise. This dataset is unique in the sense that it includes both the sovereign bond holdings and the corresponding gross CDS exposures for 65 major European banks, which are listed in table 14. To our knowledge, this paper is the first academic paper to exploit this feature of this data. Bond and CDS data are available for 28 sovereigns[8], where sovereign bond holdings are broken down by maturity and by type of holding ("held to maturity", "available for sale"

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[8] These entities are : Austria, Belgium, Bulgaria, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Latvia, Lithuania, Malta, Netherlands, Norway, Poland, Portugal, Romania, Slovakia, Slovenia, Spain, Sweden and the United Kingdom.

or "held for trading"). The dataset also includes extensive information on the capital composition of each institution and is complemented with additional public price and balance sheet information extracted from Bloomberg.

Regarding the quality of the data, one of the important features is the high degree of harmonisation across all European countries (see EBA [2011]) - meaning that consistent definitions of exposures have been used for all national banking sectors. Regarding CDS exposures, a reassuring feature of the data is that the notional amount of CDS bought and sold by the 65 sample banks represent around one half of the notional bought and sold worldwide for each reference entity. For the four countries for which we simulate a sovereign credit event, the ratios of CDS sold by EBA banks to total worldwide notional amounts (retrieved from DTCC' Trade Information Warehouse public data) are 51% for Irish CDS, 49% for Italian CDS, 56% for Portuguese CDS and 44% for Spanish CDS. Descriptive statistics on reference entities are presented in table 9. Overall, the sample represents a gross notional of 346 billion euros for CDS sold. The net notional amount sold nevertheless is much smaller, equal to 18.9 billion euros. Interestingly, the European banking groups in our sample are overall net sellers of CDS protection on European sovereigns.

4 Calibration

4.1 Sovereign exposures and capital

We use total net sovereign exposures to calibrate B_{ij} for all i and j. Net exposures differ from gross exposures in that they account for provisions. In addition to bonds, they may include loans and advances, which are also assumed to default in case of sovereign credit event. The capital K_i of each institution is calibrated as its common equity. Such a definition of capital excludes hybrid instruments, ordinary shares subscribed by governments or existing government support measures, whose value would be uncertain in case of sovereign credit event.

4.2 Bilateral CDS exposures

Our dataset contains the notional CDS exposures at a bank level, but not the full matrix of bilateral exposures. For each reference entity, we estimate such a matrix through an augmented entropy maximization method. Simple entropy maximisation has been widely used in the literature to estimate interbank loans and borrowings and to assess contagion in the absence of observed interbank lending patterns (see Upper and Worms [2004]). In contrast, it has scarcely been used to estimate bilateral CDS exposures. The ability of this method to fit actual interbank loans exposures is discussed in Mistrulli [2007].

A first step consists of approximating the share of the CDS exposure of the sample banks that is held by other sample banks. For each reference entity, we retrieve the global CDS gross notional amount from the DTCC and compute the share of which our sample banks account for. The share of the exposure of our sample banks *vis-a-vis* other sample banks is assumed to be equal to this ratio.

The entropy maximization method is not described here in full details, as a full derivation of the optimization problem can be found in Wells [2004]. It is solved numerically by using the RAS-algorithm [9]. One drawback of the entropy maximisation method is that the estimated network is not sparse as links of even small magnitude are estimated between any two institutions with a strictly positive aggregate exposure. We overcome this drawback by imposing a lower bound on the notional value of each bilateral exposure. Once the non-sparse matrix estimated by simple entropy maximization is obtained, all transactions whose notional value g_{ik}^{j} is below $\bar{g} = 5$ million euros are dropped and $g_{ik}^{j} = 0$ is imposed instead [10]. The bilateral exposure matrix is then re-estimated so that the cross-entropy between this sparse matrix and the outcome matrix is minimized, under the constraint that the bilateral buy and sell exposures sum up to their true value for each bank. This augmented method has two advantages. First, it generates sparse matrices. Second, we can run the whole simulation exercise for various values of $\bar{g}$, i.e. different densities of the CDS network for an given aggregate notional exposure. The results presented later on the number of failures and on the relative share of each failure channel are found to be robust to changes in $\bar{g}$.

4.3 CDS portfolios

Bilateral exposures on the CDS market typically result from several offsetting or reinforcing transactions. To compute the market value of each particular exposure of a bank i *vis-a-vis* its counterparty k, one needs to know the dates at which each of the transactions that make up the exposure

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[9] The RAS-Algorithm, also called iterative proportional fitting procedure, is exposed by Blien and Graef [1991].

[10] Chen et al. [2011] showed that the median CDS of a trade on the CDS market is about 8 million euros. Even though the network becomes denser when a lower cut-off threshold is imposed, our results regarding the risk propagation channels are robust and remain unchanged to a large extent when a different threshold is imposed. This is due to the fact that contagion purely due to CDS exposures is relatively limited in our simulations (see below).

have been opened, as well as their maturity. Such data is not available in the dataset and must be simulated. All CDS are assumed to have a 5-years maturity, i.e. the most common maturity on the market (see Chen et al. [2011]). Any net bilateral exposure between two banks i and k is assumed to result from multiple (potentially offsetting) trades, each of them having an average notional amount equal to 8 million euros [11]. For each reference entity, a numerical algorithm ensures that the number of transactions from which each position results and its notional value are drawn from the distributions detailed in table 1, but also that the resulting gross and net positions equal those available in the data.

Variable	**Distribution**	**Calibration**
Average transactions number	Power law	Scale = 2
Average transaction notional	Log-normal	$\mu = 8$ Mn, $\sigma = 2$ Mn
Date of transaction	Truncated Gaussian	Support $[-5; 0]$ $\mu = -1.5$ Mn, $\sigma = 1$

Table 1 – CALIBRATIONS FOR CDS PORTFOLIO SIMULATIONS

Each simulated transaction is randomly assigned a date (the time at which the CDS is bought) drawn from a truncated Gaussian density function with support $[-5; 0]$. Its mean, equal to -1.5, is such that the average remaining maturity of a contract is 3.5 years. The randomly drawn dates are then matched with particular trading days and with the corresponding price data, therefore enabling the computation of market values.

4.4 Market values of CDS portfolios

The computation of the market values of CDS portfolios is based on the valuation method exposed in O'Kane and Turnbull [2003]. At any time t_v, the market value of a CDS position is equal of the current market value of the remaining protection minus the expected present value of all premia to be paid until default or maturity, whichever is sooner. For the CDS buyer, it can be written as :

$$V(t_v, t_m) = [q\,(t_v, t_m) - q\,(t_0, t_m)] \times RPV01\,(t_v, t_m) \tag{20}$$

, where $q\,(t_0, t_m)$ is the contractual spread, $q\,(t_v, t_m)$ the spread at the time of valuation and $RPV01\,(t_v, t_m)$ the risky present value of one basis

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[11] The average notional amount of a trade is calibrated using public data provided by DTCC. On a weekly basis, DTCC releases data on the weekly transcation activity, including the gross notional amount traded and the number of trades, for 1000 reference entities.

point paid on the premium leg of the CDS contract until either default or maturity. The calculation of $RPV01\left(t_v, t_m\right)$ requires a model accounting for the probability of the reference entity surviving at each premium payment date. The term structure of arbitrage-free hazard rates is obtained from CDS of different maturities through a bootstrapping procedure.

4.5 Pledgeable assets

We consider only cash and government securities to be usable as pledgeable collateral $\bar{C}_i$. According to ISDA [2012a], these two asset classes represent far above 90% of the collateral used on OTC markets. We obtain data on banks' cash from Bloomberg. To obtain $\bar{C}_i$, haircuts have to be applied on sovereign bonds included in the pool of free collateral. Haircuts on sovereign bonds typically depend on the rating of the issuer as well as on the maturity of the pledged bond. For each sovereign entity, ratings are retrieved from Fitch Ratings as of 30 September 2011. Countries are classified by ratings into three buckets : from AA- to AAA (prime and high grade), from BB- to A+ (medium grade) and from D to BB+ (speculative grade or defaulted). Haircuts for the higher bucket (broken down by maturity) are obtained from CME [CME, 2012]. Haircuts for the medium bucket are assumed to be twice higher than haircuts for the higher bucket. Bonds in the lower bucket are assumed not to be pledgeable, in conformity with the usual market practice. Haircuts by rating and maturity are presented in table 2.

Rating range	Countries	0-5 years	5-10 years	> 10 years
AA- to AAA	AT, BE, CZ, DK, FI, FR, DE, NE, NO, SL, SP, SW, UK	6%	7,50%	9%
BBB- to A+	BU, CY, EE, HU, IR, IT, LT, LN, MT, PL, PT, RO, SK	12%	15%	18%
D to BB+	GR, IC	Not pledgeable	Not pledgeable	Not pledgeable

TABLE 2 – HAIRCUTS ON PLEDGEABLE ASSETS BY RATING AND MATURITY. Ratings are as of 30 September 2011, by Fitch Ratings.

$\bar{C}_i$ is then the sum of cash holdings plus the pledgeable value of sovereign bonds. Whereas cash is valuable as collateral in full amount, part of the sovereign bond holdings are assumed to be encumbered, i.e. pledged as collateral in other transactions (e.g. repurchase agreements, covered bonds, etc.). The ratio of asset encumbrance is assumed to be 50%.

4.6 Collateralisation level

The ISDA (see ISDA [2012b]) provides data on the average collateralization level of OTC derivatives transactions by type of counterparty. For banks and broker-dealers, the average collateralization level was 78.6% in 2011. Given that the sample consists of major European banks, we assume each τ_{ik}^{j} to be drawn out of a uniform distribution with support $[0.6; 1]$.

4.7 Tail dependences

Tail correlations of sovereign bond prices are estimated from weekly price data retrieved from Bloomberg. We retrieve prices (excluding accrued interest between coupon dates) of 5-years government bonds maturing in 2012. Therefore, we hold between 3 and 4 years-long time series ranging from the emission of a bond to the date of the stress scenarios (30 September 2011), which we use to estimate the parameters of the copula described in equation 2. The estimated correlation parameters for the t copula are presented in table 11, together with the unconditional correlations (table 10). As can be seen from the table, high correlations are estimated between the bond prices of stressed countries.

Using the same copula framework, tail dependences of sovereign CDS spreads are estimated from weekly observations of senior 5-year CDS spreads. The data is retrieved from Bloomberg for all countries except Iceland and Malta[12] spanning from October 2006 to September 2011. To estimate the tail dependence of the spreads of the CDS of the defaulted reference entity $\bar{j}$ and the other CDS, we use the longest available time series. The first difference (i.e. the CDS returns) of each series is then filtered by an ARMA-GARCH model, using an ARMA(2,2) and a GARCH(2,2) model. The residuals are then fed to the t copula. The unconditional correlation coefficients together with the estimated correlation parameters for the t copula are presented in tables 12 and 13. Once again, high CDS return correlations are to be found between stressed countries.

For the estimation of both tail correlations, we test two alternatives to the t copula, a Gaussian copula and Archimedean copulas. The pattern of estimated correlations remains unchanged to a large extent. As a robustness check, the whole set of simulations has been re-run for the three copula specifications. The main results, regarding the number of failures and the relative magnitude of each default channel, do change only marginally. The

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[12] The CDS data series for these two countries either do not exist or are too short to enable robust estimations.

difference between the results presented below and the results for alternative specification is of at most one failure[13].

4.8 Recovery rate

The recovery rate $RR_{\bar{j}}$ on the defaulted sovereign bond $\bar{j}$ is a key parameter of the theoretical framework, as it impacts directly the loss incurred on sovereign holdings but also the magnitude of jumps of both other bonds' value and of CDS spreads. Data on recovery rates are scarce due to the relatively rare occurrence of sovereign credit events. Sturzenegger and Zettelmeyer [2005] or Moody's [2012] more recently document case-specific factors leading to a high variability of recovery rates. 30-days post-default prices of bonds as a percentage of the par value during the last 15 years range from 18 in Russia (in 1998) to 95 in Dominican Republic (in 2005). Over the sovereign defaults studied by Moody's [2012], the average recovery rate lies around 53%. In the analysis, we study the relative importance of each contagion channel under a wide range of recovery rates.

5 Simulation of credit events and results

We simulate credit events of four European countries, i.e. Ireland, Italy, Portugal and Spain and we restrict ourselves to simulated jumps-to-default of one particular country at the time. The date of the stress scenarios is 30 September 2011 (due to available data[14]).

5.1 Bank failure channels

Table 3 summarizes the main results concerning the relative magnitude of each bank failure channel identified in the theoretical model, whereas tables 15 to 18 present the number of bank failures and the relative magnitude of each of the failure channels for a wide range of recovery rates.

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13. For space reasons, the results obtained with alternative specifications are not presented here. They are available upon request.

14. In October 2012, the EBA released the final results of the EU Capital Exercise, showing that the European Banks involved in the exercise had increased their capital by more than 200 billion euros between December 2011 and June 2012. At the same time, banks had increased their sovereign exposures, particularly in the countries under market stress. Unfortunately, the latter data disclosure by the EBA does not include banks' sovereign CDS positions.

	Recovery rate	**Direct sovereign loss**	**Correlated bond losses**	**Collateral shortage**	**Contagious insolvency**	**Contagious illiquidity**	**Total**
Ireland	0,1	0	10 (83%)	2 (17%)	0	0	12
	0,5	0	0	3 (100%)	0	0	3
	0,9	0	0	0	0	0	0
Italy	0,1	6 (24%)	17 (68%)	2 (8%)	0	0	25
	0,5	1 (8%)	10 (77%)	2 (15%)	0	0	13
	0,9	0	0	2 (100%)	0	0	2
Portugal	0,1	3 (43%)	2 (29%)	2 (29%)	0	0	7
	0,5	0	1 (33%)	1 (33%)	0	1 (33%)	3
	0,9	0	0	0	0	0	0
Spain	0,1	5 (21%)	17 (71%)	2 (8%)	0	0	24
	0,5	2 (15%)	9 (69%)	2 (15%)	0	0	13
	0,9	0	0	3 (100%)	0	0	3

TABLE 3 – RELATIVE MAGNITUDE OF THE BANK FAILURE CHANNELS. The table shows the number of failed banks (common equity<0), while the percentages in parentheses indicate the relative share of each failure channel. Percentages may not sum up to 1 due to rounding.

For all simulated countries, the number of bank failures and the relative importance of each bank failure channel is found to depend importantly on the recovery rate of the sovereign bonds. When the recovery rate is low, bank failures due to insolvency play a predominant role, and are mainly driven by failures due to write-downs on correlated sovereign exposures. Failures due to direct losses on sovereign bonds increase in number when the recovery rate decreases, but are limited in most cases to domestic banks (as indicated by the red figures in parentheses in tables 15 to 18). When the recovery rate increases, the relative importance of bank failures due to insolvency decreases, whereas failures due to collateral shortage become more prominent. For higher recovery rates, only a few (if any) failures of banks due to their inability to meet collateral calls are observed.

Regarding CDS-related bank failures, interestingly, we find the collateral shortage on the CDS market to be a more important vulnerability than direct CDS repayments for the settlement of contracts on the defaulted reference entity. The importance of the collateral shortage channel is magnified for banks, which have a relatively high net CDS exposure compared to their pool of liquid assets, and subsists in most scenarios even for high recovery rates. In contrast, we find that even though not all CDS repayments are paid in full, they are found not to trigger contagion. This result is discussed in further details below.

5.2 The distribution of capital ratios

Tables 19 and 20 show the percentage of banks that have respectively a ratio of common equity over risk-weighted assets below 0% and below 4.5% (i.e. undercapitalisation according to Basel III threshold). In case of a Spanish or Italian credit event with low recovery rates, up to one third of the European banking system may end up with negative equity and about two third may be under-capitalised, whereas the consequences of an Irish or Portuguese credit event are more limited. Moreover, failing or undercapitalised banks are mainly smaller banks (in terms of risk-weighted assets) as the share of defaulted banks is steadily higher than the share of defaulted assets.

How are capital shortfalls brought about ? The theoretical framework allows losses to be incurred through three different types of channels, namely *(i)* direct losses on the defaulted sovereign bonds, *(ii)* correlated losses on the non-defaulted bond exposures and *(iii)* termination losses due to counterparty failures. The decomposition of capital losses is presented in tables 21 to 24 over a wide range of recovery rates. Whereas direct losses are predominant for local banks, correlated losses are, on average, an important source of losses for foreign banks, highlighting the importance of price effects. The main explanation for the importance of direct losses on sovereign bond exposures for local banks is that they typically hold a disproportionately high share of their own sovereign bonds relative to their other sovereign exposures (home bias).

5.3 The redistributive effects of CDS

Even though CDS repayments are not the main source of bank failures stemming from the CDS market as a consequence of the simulated European sovereign credit events, analysing CDS repayments is nevertheless interesting in two respects, namely regarding *(i)* their magnitude and *(ii)* the extent of their redistributive effects.

Concerning the magnitude of CDS repayments, table 25 presents the net payables and the actual repayments at a system level for the four default scenarios and three recovery rates. Aggregate actual repayments are of low magnitude (compared to the total pool of liquid assets, which is 2.9 trillion euros), as their maximum is 2.6 billion euros (in the case of Italian sovereign credit event with 0.1 recovery rate) and rarely exceed 1 billion euros. The ratio of actual repayments over net payables increases with the recovery rate, but remains high overall (0.72 on average when the recovery rate is 0.5) so that it cannot explain the low level of actual repayments. The ratio is the smallest in case of Spanish default and the highest in case of Portuguese default.

Regarding the redistributive effects of CDS payments, two effects are analysed. First, we compute a liquidity ratio for each institution (defined as the ratio of liquid assets $\bar{C}_i$ over risk-weighted assets) and observe whether, in all pairs of banks proceeding to a strictly positive net bilateral CDS repayment, the beneficiary of the repayment has a lower liquidity ratio *ex ante* than the payer, i.e. whether CDS repayments tend to go from "high liquidity" banks to "low liquidity" banks. Second, we compute a loss ratio for each bank (defined as the ratio of direct sovereign losses incurred over risk-weighted assets) and look whether, in the same pairs of banks, repayments tend to flow from "low loss" banks to "high loss" banks.

Redistributive effect	**Ireland**	**Italy**	**Portugal**	**Spain**
From "high liquidity" to "low liquidity"	0,60	0,46	0,52	0,47
From "low loss" to "high loss"	0,52	0,63	0,51	0,49

Table 4 – REDISTRIBUTIVE EFFECTS OF CDS REPAYMENTS. The ratios correspond to the percentage of pairs of banks for which a redistributive effect is observed over the total number of pairs of banks for which a net CDS repayment exists.

Results for the four default scenarios are presented in table 4. Overall, we observe little redistributive effects, as the percentage of pairs of banks for which a redistribution is observed is close to 50%. Such result must nevertheless be interpreted cautiously, as we do not observe the full portfolio of the counterparties in the CDS market. Moreover, it may be that direct sovereign bond holdings are imperfect proxies for actual country-specific exposures, therefore that the loss ratio defined earlier might not be an ideal way to assess the true redistributive effects of CDS repayments.

5.4 Contagion

In the simulations, we find only one contagious bank failure (see table 17). Five main explanations account for the limited extent of contagion. First, our framework only captures one type of interconnections between banks, i.e. bilateral CDS exposures, and misses other important exposures, chiefly interbank exposures and other derivatives exposures. This caveat nevertheless enables us to focus on contagion purely due to banks' European sovereign exposures, and therefore to isolate and quantify the importance of this particular channel of contagion. Second, losses due to counterparty failures are of low magnitude. This can be seen from table 5, which compares banks' losses due to counterparty failures with their remaining capital after the imputation of losses on direct and correlated bond exposures. Third, collateralization and close-out netting play a risk-mitigating role (the details are explained below).

	Ireland	**Italy**	**Portugal**	**Spain**
Losses due to counterparty risk	0.2	1.5	0.1	1.1
Remaining capital	797.5	503.9	895.7	509.2

TABLE 5 – LOSSES DUE TO COUNTERPARTY RISK AND REMAINING CAPITAL (in billion euros). Remaining capital corresponds to the aggregate capital that remains in the banking sector once losses on direct and correlated sovereign bond exposures have been imputed.

The fourth reason for the low extent of the contagion is due to the network structure. A large share of the links in each estimated gross CDS network (between 52% and 86% depending on the reference entity - and a mean of 76%) are reciprocal [15], implying that potentially contagious chains of financial institutions are relatively limited.

Finally, we do not observe the default of one of the main dealers on the CDS market.

6 Robustness checks and the dynamics of the model

In this section, we present simulation results when some of the main calibration parameters or assumptions are altered to analyse the robustness of the results. Moreover, this enables us to explore the risk-mitigating role of certain collateral management schemes and of close-out netting.

6.1 Collateral agreements and the level of collateralization

Collateral netting agreements used in the theoretical framework reduce to a large extent the amount of collateral to be posted at a system level. Whereas the aggregate collateral requirement is 2.7 billion euros when netting agreements are in place, it would rise to 36.6 billion euros if they were to be suppressed. In that regard, collateral netting agreements in this setting increase the overall liquidity of the banking sector, as less cash and liquid assets have to be pledged as collateral. Such a positive role of collateral netting agreements should nonetheless be considered cautiously, as the theoretical framework does not capture strategic bank balance sheet decisions when

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[15] A link between two banks i and k exists on the reference entity j whenever $g^j_{ik} > 0$ or $g^j_{ki} > 0$ and is said to be reciprocal if $g^j_{ik} > 0$ and $g^j_{ki} > 0$.

the institutional framework changes. For example, the existence of collateral netting agreements is likely to induce a higher leverage *ex ante*, as larger derivative portfolios can be sustained with a given level of pledgeable assets.

Regarding the level of collateralization of each trade (i.e. τ_{ik}^{j}), it plays an ambiguous role in the present setup. On the one hand, collateralization reduces the extent of potential contagion by decreasing the loss incurred in case of counterparty failure. On the other hand, failures from illiquidity (i.e. inability to meet collateral calls) are more likely to occur when the required level of trade collateralization is higher, as the pool of cash and liquid assets remains constant. Up to now, we have assumed that all transactions were collateralized, but that the level of collateralization was transaction-specific. We now assume that only a fraction $\upsilon \in [0;1]$ of the deals are collateralized (with a collateralization level drawn from the same distribution as before), whereas a fraction $(1-\upsilon)$ is left uncollateralized. We focus of the dynamics of the model when υ ranges from 0 to 1. Losses due to counterparty failures for selected values of υ when the recovery rate is 0.5 are presented in table 6.

Level of collateralization (υ)	**Ireland**	**Italy**	**Portugal**	**Spain**
0	318.8	1914.9	33.5	1442.0
0.3	293.6	1790.3	20.3	1290.3
0.7	261.7	1560.5	16.5	1123.2
1	221.4	1476.4	10.4	1071.5

Table 6 – LOSSES DUE TO COUNTERPARTY FAILURES WITH DIFFERENT LEVELS OF COLLATERALIZATION (in million euros). The recovery rate is set to 0.5.

Overall, we do find limited effects of changes in the level of collateralization. Losses due to counterparty failures are higher when collateralization is lower, but there is no one-to-one relationship (meaning that doubling the number of collateralized trades does not half the losses due to counterparty failures, but by a much smaller amount - except in the case of Portugal's credit event). In general, the losses remain of low magnitude, partly due to the fact that our dataset captures only part of banks' actual derivatives portfolios. Regarding the consequences of a lower value of υ on banks' liquidity, we find only one case - when Portugal defaults - where a lower level of trade collateralization reduces the number of bank failures due to banks' inability to deliver eligible collateral. It is important to stress, once again, that such a result does not account for strategic balance decisions of banks in a dynamic setting, where a lower level of collateralization *ex ante* may induce banks to take on more leverage and make the whole system more vulnerable.

6.2 Close-out netting

In order to assess the extent to which close-out netting mechanisms mitigate bank-to-bank contagion in case of counterparty failure, we test the baseline specification of the theoretical framework (section 4) in an environment where close-out netting would not apply. This implies that, when a bank k fails, each of the bilateral derivative exposures between k and its non-failed counterparties i is considered as a separate asset or liability. More precisely, all CDS positions that were in-the-money for k are considered as *immediately payable* liabilities (equal to the market value of the position) for the non-failed party i. Similarly, positions which were out-of-the money for k (therefore in-the-money for i) are assets for i. But because k failed, the payments of these assets may be delayed for months or years and only a part of it can be recovered. Testing for the consequences of such a framework compared to an environment where close-out netting is implemented enables assessing the risk-mitigating role of the close-out netting.

Solving for the contagion process in such a framework is a problem similar to the one studied by Eisenberg and Noe [2001]. We make use of their clearing payment vector approach to solve for the equilibrium number of failures. Using a fixed-point argument, they show the existence of a unique clearing payment vector in a system of institutions mutually interconnected through assets and liabilities, where banks can become insolvent if the value of their liabilities rises above those of their assets. Moreover, an attractive feature of this algorithm is that it satisfies both limited liability of banks and proportional sharing of the recovery value in the case of failure of a bank.

	Ireland	**Italy**	**Portugal**	**Spain**
Fundamental failures	5	15	2	12
Contagious failures with close-out netting	0	0	1	0
Contagious failures without close-out netting	1	30	0	30
Total failures with close-out netting	5	15	3	12
Total failures without close-out netting	6	45	2	42
Share of failed assets with close-out netting	0,03	0,17	0.02	0,17
Share of failed assets without close-out netting	0,04	0,88	0.01	0,89

Table 7 – THE NUMBER OF BANK FAILURES WITHOUT CLOSE-OUT NETTING. Columns denote countries for which a sovereign credit event is simulated. The recovery rate is set to 0.5.

We use the Eisenberg and Noe [2001] algorithm to clear the network of sovereign CDS exposures as a consequence of fundamental failures. The results of the simulations are presented in table 7 for a recovery rate equal to 0.5. We observe that in contrast to the situation where close-out netting is enforced, contagious failures may be substantial when it does not exist.

This is even more true for the default scenarios of Italy and Spain, where a large share of banks exposed to the CDS market is driven to failure (45 and 42 bank fail respectively). Interestingly, and contrary to what is observed when close-out netting is introduced, we do observe the failure of some or all the main dealers when close-out netting does not exist. This is reflected in the very large share of failed assets at a system level (defined as the ratio of assets of the *ex post* failing banks over the *ex ante* total assets in the system), which reaches 88% following a failure of Italy and 89% following a failure of Spain. One exception is Portugal, where we find one contagious failure with close-out netting but zero failure without. This result highlights the major role played by close-out netting to limit contagion and the importance of the well-functioning of this very process.

7 Conclusion

This paper models credit events and their spillovers to the European banking system. It provides a theoretical framework to assess the potentially risk-mitigating or risk-amplifying role of the CDS market in case of a simulated sovereign credit event. Rather than focusing purely on CDS exposures, it concentrates on the interplay between banks' sovereign bond and CDS holdings, therefore enabling an in-depth analysis of credit risk transfer mechanisms. One characteristic of the theoretical framework is that it incorporates several features proper to OTC derivatives markets.

Based on the theoretical model that the paper develops, we are able to document the relative magnitude of five bank failure channels for a wide range of recovery rates. According to our simulation results using the EBA 2011 EU Capital Exercise data for 65 major European banks, overall, losses due to sovereign bond exposures appear to be significantly more important in magnitude than losses due to pure CDS exposures and to counterparty risk on the CDS market. However, for all simulated countries, the number of bank failures and the relative importance of each bank failure channel is found to depend importantly on the recovery rate of the sovereign bonds. When the recovery rate is low, bank failures due to insolvency play a predominant role, and are mainly driven by failures due to write-downs on correlated sovereign exposures. Failures due to direct losses on sovereign bonds increase in number when the recovery rate decreases, but are limited in most cases to domectic banks. When the recovery rate increases, the relative importance of bank failures due to insolvency decreases, whereas failures due to collateral shortage become more prominent. For higher recovery rates, only a few (if any) failures of banks due to their inability to meet collateral calls are observed.

Concerning results related to the the sovereign CDS market, even though the observed distribution of net protection bought through CDS does not match the distribution of underlying sovereign bond holdings, we do not find significant failures due to the inability of some banks to honour their contractual repayments in case of simulated sovereign credit event. Overall, CDS repayments remain at low levels compared to banks' liquid asset pools and to capital ratios. In this regard, the usual focus - at least in the financial press - on the large (gross) amounts at stake on the CDS market might be misleading, as it occults another more important source of fragility. According to our results, the largest source of vulnerability for the CDS protection sellers is found to be the sudden increases in collateral to be posted, i.e. the inability of financial institutions to meet collateral calls. Paradoxically, whereas collateral posting and variation margins are counterparty risk mitigation mechanisms, they can turn out to be major drivers of counterparty failures at times of elevated financial stress, i.e. when collateral has to be delivered on multiple positions at the same time.

Finally, in our simulations, we do not find significant contagion purely due to the failure of counterparties on the CDS market. Explanations include the fact that several interconnectedness channels are ignored (e.g. interbank loans and deposits), that banks failing from direct or correlated bond losses are mainly medium-size institutions with little activity on the CDS market and that the default of a major dealer is never observed in the simulations. Finally, close-out netting of the whole CDS portfolio in case of counterparty failure is shown to play a major risk-mitigating role, as contagion would affect most of the banks active on the CDS market if it were not to be implemented.

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8 Appendices

8.1 Stylized facts

Although the EBA dataset on banks' CDS exposures has been made public, it has scarcely been used by the academic community, and some of its key features have remained unnoticed. We shall focus on the joint distribution of sovereign bond and CDS holdings. More precisely, if CDS are primarily used for hedging credit risk, one might expect to observe a strong correlation between sovereign bonds held and net CDS protection bought. In this subsection, we document *(i)* the statistical relation between sovereign bond holdings and net CDS bought, *(ii)* the extent to which potentially naked CDS protection is bought and *(iii)* the extent to which wrong-way risk may occur.

We first compute Pearson correlations across all reference entities between sovereign bond holdings and net CDS protection bought. We keep only the pairs of observations for which both are strictly positive[16]. The correlation is found to be positive but low (equal to 0.25), as shown in table 8. A low correlation is robust to several alternative specifications for the sovereign bond holdings. It might indeed be that banks hedge through CDS only their exposures in the held-to-maturity book, whereras available-for-sale positions are left unhedged. Or, alternatively, they might hedge only exposures with maturities above some threshold, leaving short-term exposures unhedged. Testing such specifications, we do not find high correlations. In particular, the correlation between net CDS protection bought and bonds held-to-maturity is not significantly different from zero at a 10% significance level.

	All bond holdings > 0	**HTM bonds holdings** > 0	**Bond holdings over 5 years** > 0
Net protection bought > 0	0,25	0,17	0,21
	(0,001)	(0,15)	(0,008)
Net protection bought ≥ 0	0,03	-0,02	0,01
	(0,37)	(0,72)	(0,81)

TABLE 8 – CORRELATIONS OF SOVEREIGN BONDS AND CDS BOUGHT. p-values are reported in parentheses. "HTM bonds" refers to bonds held-to-maturity. Holdings over 5 years refer to the sovereign bond holdings with average remaining maturity of higher than 5 years.

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16. Keeping the pair of observations for which bond holdings is strictly positive and net CDS bought positive or zero, one finds correlation coefficients close to zero, and statistically not different from zero. This alternative specification of the sample is presented in table 8

Second, we aim at documenting the extent to which potentially naked CDS positions are held. Naked CDS activities can be quantified according to two definitions. First, in a restrictive sense, we consider as naked CDS trading the behaviour of an institution which is a net buyer of CDS on a reference entity for which it does not hold sovereign bonds *at all.* According to this definition, 51.8% of all net protection bought is naked (in terms of number of transactions[17]). 24 banks have at least one naked CDS position. Second, in a broader sense, naked CDS trading can also be considered as an excess of net protection bought over actual bond holdings. Considering such definition, 71.5% of all net buying positions are to be considered as naked, either partially or in full. Several explanations can be proposed for the low correlation between sovereign bonds and CDS holdings. First, it can be that CDS are used mainly for trading rather than for actual hedging purposes. Second, it may be that they are not used to hedge against a particular sovereign bond *per se*, but for broader *macro hedging* purposes (i.e. for debt exposures that are not sovereign, but positively correlated with sovereign risk factors and for which no CDS exists - such a railway companies). We are not able to document such strategies, as we do not hold data on the full credit portfolio of banks. The present paper does not aim at providing an explanation for potentially naked exposures, as it does not model the incentives of banks to hold CDS.

Finally, an important issue for our purposes is the one of *wrong-way risk*, which occurs when the probability of default of a reference entity is positively correlated with the probability of a default of its net protection seller. On the CDS market, one usually considers wrong-way risk to arise when an institution is a net seller of the CDS of the sovereign of its home country. In the EBA sample, we find 14 institutions (i.e. roughly one third of the active banks) being net sellers of the CDS of the sovereign of their home countries. A large share of the wrong-way protection selling is performed by German banks (6 banks), whose home-country sovereign risk is typically one of the lowest in Europe. Concerning the countries under stress, 3 Italian banks but none of the Spanish banks are net sellers of their home sovereign CDS.

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[17] In terms of notional amount, naked protection bought accounts for 51.5% of all net protection bought.

8.2 Appendix. Descriptive statistics - Tables

Statistics	**Sample**
Number of banks	65
Number of reference entities	28
Gross notional bought - all refs.	327.6 Bns
Gross notional sold - all refs.	346.5 Bns
Net notional sold	28.2 Bns
Ireland	
Gross notional sold	11.9 Bns
Gross notional bought	11.3 Bns
Italy	
Gross notional sold	83.6 Bns
Gross notional bought	78.4 Bns
Portugal	
Gross notional sold	20.6 Bns
Gross notional bought	20.1 Bns
Spain	
Gross notional sold	40.3 Bns
Gross notional bought	38.8 Bns

TABLE 9 – DESCRIPTIVE STATISTICS Source : EBA 2011 EU-wide Capital Exercise. Ireland, Italy, Portugal and Spain are the four countries for which we simulate a jump-to-default credit event.

	AT	**BE**	**CZ**	**DK**	**FI**	**FR**	**DE**	**IR**	**IT**	**NE**	**NO**	**PL**	**PT**	**SP**	**SW**	**UK**
IR	0,13	0,35	0,04	-0,09	0,01	0,10	0,05	0	0,54	0,01	-0,05	-0,03	0,65	0,53	-0,09	-0,06
IT	0,53	0,71	0,34	0,10	0,32	0,52	0,40	0,54	0,00	0,33	0,07	-0,01	0,44	0,77	0,04	-0,02
PT	0,14	0,28	0,00	-0,05	0,07	0,11	0,07	0,65	0,44	0,08	-0,06	0,00	.	0,47	-0,04	0,03
SP	0,49	0,67	0,29	0,09	0,31	0,49	0,38	0,53	0,77	0,29	0,08	-0,08	0,47	.	0,05	0,13

TABLE 10 – UNCONDITIONAL CORRELATIONS OF SOVEREIGN BOND RETURNS. Estimated from weekly 5-year government bond returns. Data source : Bloomberg.

	AT	**BE**	**CZ**	**DK**	**FI**	**FR**	**DE**	**IR**	**IT**	**NE**	**NO**	**PL**	**PT**	**SP**	**SW**	**UK**
IR	0,21	0,44	0,01	-0,09	0,09	0,16	0,07	.	0,59	0,09	-0,06	0,09	0,63	0,58	-0,07	-0,13
IT	0,45	0,69	0,23	0,05	0,25	0,43	0,27	0,59	0,00	0,26	.	-0,02	0,46	0,78	0,04	-0,07
PT	0,20	0,36	-0,08	-0,04	0,15	0,16	0,10	0,63	0,46	0,15	-0,04	0,01	.	0,48	0,00	0,02
SP	0,53	0,71	0,25	0,12	0,32	0,51	0,37	0,58	0,78	0,33	0,05	-0,06	0,48	.	0,07	0,11

Table 11 – ESTIMATED CORRELATIONS OF SOVEREIGN BOND RETURNS WITH THE T COPULA. Estimated from weekly 5-year government bond returns. Data source : Bloomberg.

	AT	**BE**	**CZ**	**DK**	**FI**	**FR**	**DE**	**IR**	**IT**	**NE**	**NO**	**PL**	**PT**	**SP**	**SW**	**UK**
IR	-0,27	-0,11	-0,39	-0,18	-0,17	-0,04	-0,31	.	0,04	-0,24	-0,18	-0,39	0,17	0,16	-0,11	0,21
IT	0,00	0,65	0,00	0,12	-0,04	0,27	0,03	0,04	.	0,11	-0,01	0,09	0,55	0,57	0,10	0,21
PT	-0,02	0,54	0,01	0,00	0,08	0,39	0,05	0,17	0,55	-0,05	-0,06	0,30	.	0,74	0,06	0,28
SP	0,06	0,66	0,02	0,03	0,18	0,48	0,05	0,16	0,57	0,04	-0,02	0,44	0,74	.	0,12	0,37

Table 12 – UNCONDITIONAL CORRELATIONS OF SOVEREIGN CDS RETURNS. Estimated from weekly 5-year CDS returns. Data source : Bloomberg.

	AT	**BE**	**CZ**	**DK**	**FI**	**FR**	**DE**	**IR**	**IT**	**NE**	**NO**	**PL**	**PT**	**SP**	**SW**	**UK**
IR	-0,09	-0,16	-0,24	-0,32	-0,13	-0,03	-0,20	.	0,04	-0,36	-0,33	-0,34	0,22	0,10	-0,12	0,23
IT	-0,04	0,54	0,01	0,15	-0,03	0,24	-0,01	0,04	.	0,14	-0,02	0,10	0,51	0,40	0,18	0,22
PT	-0,14	0,50	0,00	0,02	0,06	0,48	0,04	0,22	0,51	-0,03	-0,09	0,51	.	0,82	0,26	0,38
SP	0,00	0,56	0,04	0,03	0,17	0,52	0,08	0,10	0,40	0,01	-0,02	0,57	0,82	.	0,36	0,41

Table 13 – ESTIMATED CORRELATIONS OF SOVEREIGN CDS RETURNS WITH THE T COPULA. Estimated from weekly 5-year CDS returns. Data source : Bloomberg.

Country	Bank	Country	Bank
AT	ERSTE GROUP	FR	SOCIETE GENERALE
AT	RAIFFEISEN ZENTRALBANK	UK	ROYAL BANK OF SCOTLAND
AT	OESTERREICHISCHE VOLKSBANK	UK	HSBC HOLDINGS
BE	DEXIA	UK	BARCLAYS
BE	KBC BANK	UK	LLOYDS BANKING GROUP
CY	MARFIN POPULAR BANK	HU	OTP BANK NYRT.
CY	BANK OF CYPRUS	IE	ALLIED IRISH BANKS
DE	DEUTSCHE BANK	IE	BANK OF IRELAND
DE	COMMERZBANK	IE	IRISH LIFE AND PERMANENT
DE	LANDESBANK BADEN-WURTTEMBERG	IT	INTESA SANPAOLO
DE	DZ BANK	IT	UNICREDIT
DE	BAYERISCHE LANDESBANK	IT	BANCA MONTE DEI PASCHI DI SIENA
DE	NORDDEUTSCHE LANDESBANK	IT	BANCO POPOLARE
DE	HYPO REAL ESTATE HOLDING	IT	UNIONE DI BANCHE ITALIANE
DE	WESTLB AG	LU	BANQUE ET CAISSE DEPARGNE DE LETAT
DE	HSH NORDBANK	MT	BANK OF VALLETTA
DE	LANDESBANK HESSEN-THURINGEN	NL	ING BANK
DE	LANDESBANK BERLIN	NL	RABOBANK NEDERLAND
DE	DEKABANK	NL	ABN AMRO BANK
DE	WGZ BANK	NL	SNS BANK
DK	DANSKE BANK	NO	DNB NOR BANK
DK	JYSKE BANK	PL	POWSZECHNA BANK
DK	SYDBANK	PT	CAIXA GERAL DE DEPOSITOS
DK	NYKREDIT	PT	BANCO COMERCIAL PORTUGUES
SP	BANCO SANTANDER	PT	ESPIRITO SANTO FINANCIAL GROUP
SP	BANCO BILBAO VIZCAYA ARGENTARIA	PT	BANCO BPI
SP	BFA-BANKIA	SE	NORDEA BANK
SP	CAJA DE AHORROS Y PENSIONES DE BARCELONA	SE	SKANDINAVISKA ENSKILDA BANKEN
SP	BANCO POPULAR ESPANOL	SE	SVENSKA HANDELSBANKEN
FI	OP-POHJOLA GROUP	SE	SWEDBANK
FR	BNP PARIBAS	SI	NOVA LJUBLJANSKA BANKA
FR	CREDIT AGRICOLE	SI	NOVA KREDITNA BANKA MARIBOR
FR	BPCE		

Table 14 – SAMPLE OF BANKS SORTED BY THEIR HOME COUNTRY. Source : EBA 2011 EU-wide Capital Exercise.

8.3 Appendix. Results - Tables and figures

Recovery rate	**Direct sovereign loss**	**Correlated bond losses**	**Collateral shortage**	**Contagious insolvency**	**Contagious illiquidity**	**Total**
0	0	10 (2)	2	0	0	12 (2)
0,1	0	7 (2)	2	0	0	9 (2)
0,2	0	5 (2)	2	0	0	7 (2)
0,3	0	2	2	0	0	4
0,4	0	1	3	0	0	4
0,5	0	0	3	0	0	3
0,6	0	0	2	0	0	2
0,7	0	0	2	0	0	2
0,8	0	0	1	0	0	1
0,9	0	0	0	0	0	0

Table 15 – FAILURE CHANNELS - IRELAND The number of failed banks (bank capital becomes negative, $K < 0$) by failure channels and recovery rates in case of simulated Irish sovereign credit event. Red figures in parentheses indicate the number of domestic banks among the total number of failing banks. The absence of parentheses indicates that all failing banks through one channel are foreign banks.

Recovery rate	**Direct sovereign loss**	**Correlated bond losses**	**Collateral shortage**	**Contagious insolvency**	**Contagious illiquidity**	**Total**
0	7 (5)	17	2	0	0	26 (5)
0,1	6 (4)	17 (1)	2	0	0	25 (5)
0,2	6 (4)	14 (1)	2	0	0	22 (5)
0,3	4 (3)	12 (2)	2	0	0	18 (5)
0,4	3 (3)	11 (2)	2	0	0	16 (5)
0,5	1 (1)	10 (3)	2	0	0	13 (4)
0,6	1 (1)	7 (3)	2	0	0	10 (4)
0,7	1 (1)	2	3 (1)	0	0	6 (2)
0,8	0	1 (1)	3	0	0	4 (1)
0,9	0	0	2	0	0	2

Table 16 – FAILURE CHANNELS - ITALY The number of failed banks (bank capital becomes negative, $K < 0$) by failure channels and recovery rates in case of simulated Italian sovereign credit event. Red figures in parentheses indicate the number of domestic banks among the total number of failing banks. The absence of parentheses indicates that all failing banks through one channel are foreign banks.

Recovery rate	**Direct sovereign loss**	**Correlated bond losses**	**Collateral shortage**	**Contagious insolvency**	**Contagious illiquidity**	**Total**
0	3 (3)	3 (1)	2	0	0	8 (4)
0,1	3 (3)	2 (1)	2	0	0	7 (4)
0,2	2 (2)	2 (2)	3	0	0	7 (4)
0,3	1 (1)	3 (3)	3	0	0	7 (4)
0,4	1 (1)	2 (2)	2	0	0	5 (3)
0,5	0	1 (1)	1	0	1	3 (1)
0,6	0	0	1	0	0	1
0,7	0	0	0	0	0	0
0,8	0	0	0	0	0	0
0,9	0	0	0	0	0	0

Table 17 – FAILURE CHANNELS - PORTUGAL The number of failed banks (bank capital becomes negative, $K < 0$) by failure channels and recovery rates in case of simulated Portuguese sovereign credit event. Red figures in parentheses indicate the number of domestic banks among the total number of failing banks. The absence of parentheses indicates that all failing banks through one channel are foreign banks.

Recovery rate	**Direct sovereign loss**	**Correlated bond losses**	**Collateral shortage**	**Contagious insolvency**	**Contagious illiquidity**	**Total**
0	5 (5)	20	2	0	0	27 (5)
0,1	5 (5)	17	2	0	0	24 (5)
0,2	4 (4)	14 (1)	2	0	0	20 (5)
0,3	4 (4)	11 (1)	3	0	0	18 (5)
0,4	3 (3)	11 (2)	3	0	0	17 (5)
0,5	2 (2)	9 (3)	2	0	0	13 (5)
0,6	0	8 (4)	3	0	0	11 (4)
0,7	0	4 (2)	2	0	0	6 (2)
0,8	0	0	3	0	0	3
0,9	0	0	3	0	0	3

TABLE 18 – FAILURE CHANNELS - SPAIN The number of failed banks (bank capital becomes negative, $K < 0$) by failure channels and recovery rates in case of simulated Spanish sovereign credit event. Red figures in parentheses indicate the number of domestic banks among the total number of failing banks. The absence of parentheses indicates that all failing banks through one channel are foreign banks.

	Recovery rate	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
Ireland	% Banks	0.15	0.11	0.08	0.03	0.02	0.00	0.00	0.00	0.00	0.00
		(0.03)	(0.03)	(0.03)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
	% Assets	0.07	0.04	0.02	0.01	0.01	0.00	0.00	0.00	0.00	0.00
		(0.01)	(0.01)	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Italy	% Banks	0.37	0.35	0.31	0.25	0.22	0.17	0.12	0.05	0.02	0.00
		(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.06)	(0.06)	(0.02)	(0.02)	(0.00)
	% Assets	0.32	0.31	0.27	0.18	0.16	0.13	0.11	0.02	0.01	0.00
		(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.09)	(0.09)	(0.01)	(0.01)	(0.00)
Portugal	% Banks	0.09	0.08	0.06	0.06	0.05	0.02	0.00	0.00	0.00	0.00
		(0.06)	(0.06)	(0.06)	(0.06)	(0.05)	(0.02)	(0.00)	(0.00)	(0.00)	(0.00)
	% Assets	0.03	0.02	0.02	0.02	0.02	0.01	0.00	0.00	0.00	0.00
		(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)
Spain	% Banks	0.38	0.34	0.28	0.23	0.22	0.17	0.12	0.06	0.00	0.00
		(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.06)	(0.03)	(0.00)	(0.00)
	% Assets	0.35	0.29	0.21	0.19	0.19	0.16	0.09	0.06	0.00	0.00
		(0.13)	(0.13)	(0.13)	(0.13)	(0.13)	(0.13)	(0.07)	(0.05)	(0.00)	(0.00)

TABLE 19 – FAILED BANKS AND ASSETS (bank capital becomes negative, $K < 0$). % Banks refers to the percentage of defaulted banks. % Assets refers to the share of the assets held by these failed banks.

	Recovery rate	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
Ireland	% Banks	0.46	0.38	0.35	0.31	0.23	0.20	0.11	0.05	0.03	0.03
		(0.05)	(0.05)	(0.05)	(0.03)	(0.02)	(0.02)	(0.00)	(0.00)	(0.00)	(0.00)
	% Assets	0.39	0.28	0.26	0.25	0.11	0.09	0.06	0.02	0.01	0.01
		(0.02)	(0.02)	(0.02)	(0.01)	(0.01)	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)
Italy	% Banks	0.65	0.63	0.55	0.54	0.45	0.43	0.38	0.20	0.14	0.06
		(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.05)	(0.03)
	% Assets	0.69	0.68	0.53	0.52	0.43	0.42	0.33	0.16	0.11	0.03
		(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.10)	(0.06)	(0.02)
Portugal	% Banks	0.22	0.20	0.18	0.17	0.09	0.09	0.09	0.05	0.03	0.03
		(0.06)	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)	(0.02)	(0.00)	(0.00)
	% Assets	0.15	0.10	0.09	0.08	0.03	0.03	0.03	0.01	0.01	0.01
		(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)	(0.01)	(0.00)	(0.00)
Spain	% Banks	0.65	0.63	0.62	0.52	0.45	0.40	0.31	0.23	0.15	0.05
		(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.08)	(0.02)
	% Assets	0.69	0.64	0.63	0.49	0.40	0.37	0.27	0.20	0.16	0.03
		(0.13)	(0.13)	(0.13)	(0.13)	(0.13)	(0.13)	(0.13)	(0.13)	(0.13)	(0.02)

Table 20 – UNDERCAPITALISED BANKS AND ASSETS (bank capital is below Basel III requirement for common equity, $K < 0.045$). % Banks refers to the percentage of undercapitalised banks. % Assets refers to the share of the assets held by these undercapitalised banks.

Recovery rate	**Banks**	**Direct losses**	**Correlated losses**	**Termination losses**
0.1	Domestic	0.51	0.49	0
	Foreign	0.05	0.95	<0.01
0.5	Domestic	0.52	0.48	0
	Foreign	0.06	0.94	<0.01
0.9	Domestic	0.66	0.33	0
	Foreign	0.02	0.98	0

Table 21 – DECOMPOSITION OF LOSSES - IRELAND, (Percent). The table shows the share of bank losses due to direct losses on the defaulted sovereign bonds, losses on correlated sovereign bond exposures, and termination losses due to counterparty failures in case of simulated sovereign credit of Ireland. The table is arranged by recovery rate and by the domicile of the bank.

Recovery rate	**Banks**	**Direct losses**	**Correlated losses**	**Termination losses**
0.1	Domestic	0.55	0.45	0
	Foreign	0.12	0.87	<0.01
0.5	Domestic	0.55	0.45	0
	Foreign	0.12	0.88	<0.01
0.9	Domestic	0.54	0.46	0
	Foreign	0.13	0.86	<0.01

Table 22 – DECOMPOSITION OF LOSSES - ITALY, (Percent). The table shows the share of bank losses due to direct losses on the defaulted sovereign bonds, losses on correlated sovereign bond exposures, and termination losses due to counterparty failures in case of simulated sovereign credit of Italy. The table is arranged by recovery rate and by the domicile of the bank.

Recovery rate	**Banks**	**Direct losses**	**Correlated losses**	**Termination losses**
0.1	Domestic	0.58	0.42	0
	Foreign	0.05	0.95	<0.01
0.5	Domestic	0.75	0.25	0
	Foreign	0.11	0.89	<0.01
0.9	Domestic	0.56	0.44	0
	Foreign	0.08	0.92	<0.01

Table 23 – DECOMPOSITION OF LOSSES - PORTUGAL, (Percent). The table shows the share of bank losses due to direct losses on the defaulted sovereign bonds, losses on correlated sovereign bond exposures, and termination losses due to counterparty failures in case of simulated sovereign credit of Portugal. The table is arranged by recovery rate and by the domicile of the bank.

Recovery rate	**Banks**	**Direct losses**	**Correlated losses**	**Termination losses**
0.1	Domestic	0.58	0.42	0
	Foreign	0.06	0.94	<0.01
0.5	Domestic	0.57	0.43	0
	Foreign	0.06	0.94	<0.01
0.9	Domestic	0.57	0.43	0
	Foreign	0.06	0.94	<0.01

Table 24 – DECOMPOSITION OF LOSSES - SPAIN, (Percent). The table shows the share of bank losses due to direct losses on the defaulted sovereign bonds, losses on correlated sovereign bond exposures, and termination losses due to counterparty failures in case of simulated sovereign credit of Spain. The table is arranged by recovery rate and by the domicile of the bank.

	Recovery rate	**Total net exposure**	**Net payable**	**Actual repayments**	**Repayments / Payable**
Ireland	0.1	848	763	569	0.75
	0.5	848	424	321	0.76
	0.9	848	84	84	1.00
Italy	0.1	4772	4295	2601	0.61
	0.5	4772	2386	1626	0.68
	0.9	4772	477	458	0.96
Portugal	0.1	1462	1316	1149	0.87
	0.5	1462	731	639	0.87
	0.9	1462	146	146	1.00
Spain	0.1	2375	2138	910	0.43
	0.5	2375	1188	658	0.55
	0.9	2375	237	229	0.97

TABLE 25 – AGGREGATE CDS REPAYMENTS (Mn euros). Net payable corresponds to the total net exposure multiplied by the loss given default.